# Section 1.1a – Number Systems Refresher

**This booklet belongs to:**_____**Block:** _____

**Natural Numbers:** $\{1, 2, 3 \dots\}$

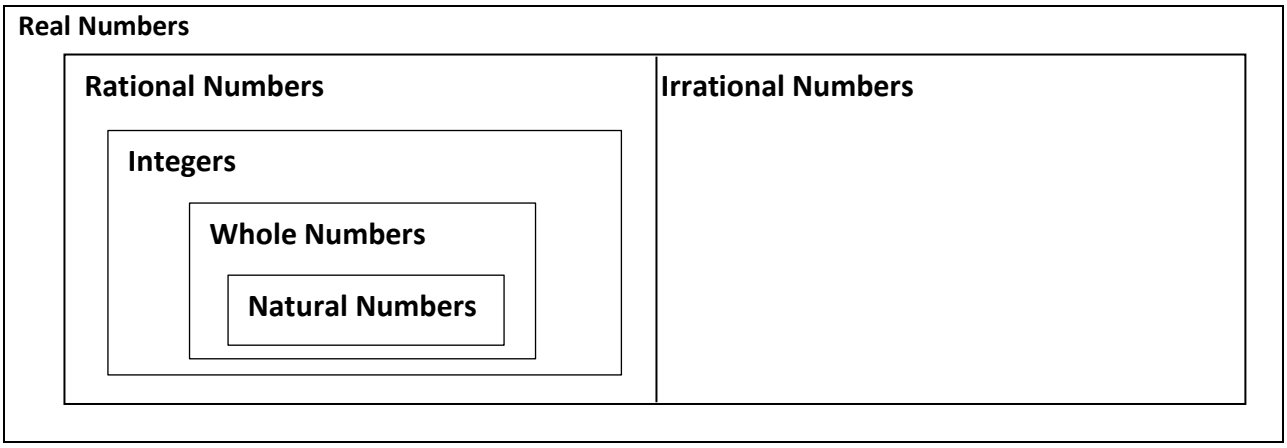
**Whole Numbers:** $\{0, 1, 2, 3 \dots\}$

**Integers:** $\{\dots, -3, -2, -1, 0, 1, 2, 3 \dots\}$

**Rational Numbers:** All numbers that **can be written as a fraction** with the denominator not equal to zero

**Irrational Numbers:** All the numbers that **cannot be written as a fraction, a terminating decimal, or a repeating decimal**

**Real Numbers:** All the rational and irrational numbers combined.




## Divisibility Properties:

Divisible by 2: A whole number is divisible by 2 if its last digit is 0, 2, 4, 6, or 8

Divisible by 3: A whole number is divisible by 3 if the sum of its digits are divisible by 3

Divisible by 4: A whole number is divisible by 4 if the last two digits are divisible by 4

Divisible by 5: A whole number is divisible by 5 if the last digit is 0 or 5

Divisible by 6: A whole number is divisible by 6 if it is an even number that is divisible by 3

Divisible by 9: A whole number is divisible by 9 if the sum of its digits are divisible by 9

Divisible by 10: A whole number is divisible by 10 if it ends in a 0

## Prime Numbers and Prime Factors

- Consider two types of whole numbers called **prime numbers** and **composite numbers**
- A **prime number** is a whole number that has exactly **2 factors**: 1 and itself
- A **composite number** is a whole number greater than 1 that has a divisor other than itself, or in other words, is not prime. Every composite number is made up of a **product of purely prime factors.**

### Here is a list of the prime numbers less than 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

**ZERO AND ONE!!**

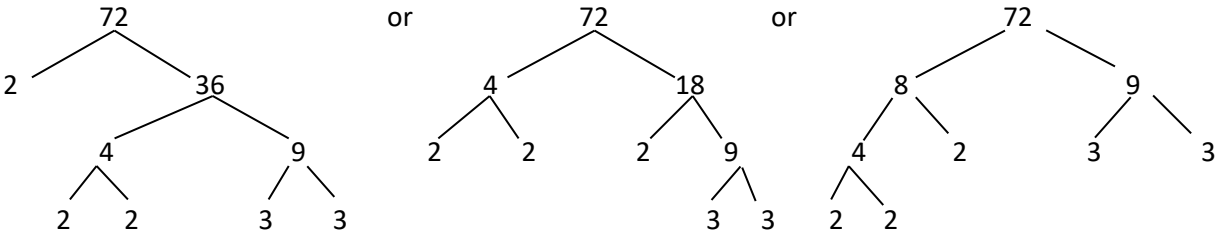
The whole numbers 0 and 1 are neither prime nor composite, why?

- Zero is not a prime factor because it has an infinite number of divisors
- One is not a prime number because it does not have two different positive whole number divisors

## Finding Prime Factors of Composite Numbers

- There are multiple methods in breaking down a number into its **factors**
- My preferred method is a **prime factor tree**
- Break the number down so that **every branch has a prime factor** on it.

**Factor tree – It doesn't matter how you start; the result should always be the same**




Therefore, the Prime Factors of 72 are: $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

- Finding the Prime Factor or just factor breakdown of numbers can be beneficial
- We can use it to:
    - Simplify Fractions
    - Remove Common Factors
    - Find Lowest Common Multiples
    - Find Greatest Common Factors
    - Simplify Radicals

**Example 1:** Find the Greatest Common Factor and Lowest Common Multiple of: $36, 48, 60$

**Solution 1:**

$36 = 2 \cdot 2 \cdot 3 \cdot 3 \quad = 2^2 \cdot 3^2$
$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \quad = 2^4 \cdot 3$
$60 = 2 \cdot 2 \cdot 3 \cdot 5 \quad = 2^2 \cdot 3 \cdot 5$

| **What Factors are Common to all?** | **The Lowest Common Multiple has to have the factors of each number in its breakdown.** |
|---|---|
| $2^2 \; and \; 3$ | I need: $2^4, 3^2, and \; 5$ |
| The GCF is: $12$ | The LCM is: $2^4 \cdot 3^2 \cdot 5 = 720$ |

**Example 2:** Find the Greatest Common Factor and Lowest Common Multiple of: $21, 28, 63$

**Solution 2:**

$21 = 3 \cdot 7 \quad = 3 \cdot 7$
$28 = 2 \cdot 2 \cdot 7 \quad = 2^2 \cdot 7$
$63 = 3 \cdot 3 \cdot 7 \quad = 3^2 \cdot 7$

| **What Factors are Common to all?** | **The Lowest Common Multiple has to have the factors of each number in its breakdown.** |
|---|---|
| $7$ | I need: $2^2, 3^2, and \; 7$ |
| The GCF is: $7$ | The LCM is: $2^2 \cdot 3^2 \cdot 7 = 252$ |

# Section 1.1a – Practice Questions

## Emerging Level Questions

1. Consider the list of numbers: $-12, \quad -2.7, \quad 0, \quad \frac{2}{3}, \quad \pi, \quad 4.21, \quad 50.$ **List all:**

| | |
|---|---|
| a) Natural Numbers | b) Whole Numbers |
| c) Integers | d) Rational Numbers |
| e) Irrational Numbers | f) Real Numbers |

## Proficient Level Questions

2. Consider the list of numbers: $-4, -0.3, 0, 0.121121112 \ldots, 2.3535 \ldots, 12, \sqrt{10}.$ **List all:**

| | |
|---|---|
| a) Natural Numbers | b) Whole Numbers |
| c) Integers | d) Rational Numbers |
| e) Irrational Numbers | f) Real Numbers |

## Extending Level Questions

3. Consider the list of numbers: $-\sqrt{64}$, $\sqrt[3]{64}$, $\sqrt[3]{0.008}$, $-\sqrt{\frac{4}{9}}$, $\sqrt{0.04}$, $\sqrt{0.4}$, $\frac{0}{\sqrt{9}}$. **List all:**

| | |
|---|---|
| a) Natural Numbers | b) Whole Numbers |
| c) Integers | d) Rational Numbers |
| e) Irrational Numbers | f) Real Numbers |

## Emerging Level Questions

4. List every number system that the following number belongs to. (Natural, Whole, etc.)

| | |
|---|---|
| a) $\sqrt{16}$ | b) $\pi$ |
| c) $0$ | d) $2.34$ |
| e) $4.010010001$ …. | f) $\sqrt{0.0004}$ |
| g) $\sqrt{\frac{27}{12}}$ | h) $-3.181818$ … |

5. List each of the following:

| | |
|---|---|
| a) Natural Numbers less than $4$ | b) Natural numbers greater than $5$ |
| c) Whole Numbers less than 2 | d) Integers greater than $-3$ |
| e) Positive integers greater than $-4$ | f) Whole numbers less than 0 |
| g) Non-negative integers less than 4 | h) Non-positive integers greater than $-4$ |

**Proficient Level Questions**

6. Without a calculator state whether the following numbers and how you know:
$63, 126, 280, 396, 575, 2610, 7800$ are divisible by:

| | |
|---|---|
| a) 2 | b) 3 |
| c) 4 | d) 5 |

e) 6

f) 9

g) 10

7. Decide whether the number is prime or composite. If it's composite, factor it into primes.

a) 19

b) 51

c) 87

d) 101

| | |
|---|---|
| e) 117 | f) 199 |
| g) 611 | h) 997 |
| i) 629 | j) 551 |

8. Simplify by using prime factorization and canceling out common factors

a) $\frac{385}{455}$

b) $\frac{1155}{1188}$

c) $\frac{1848}{2310}$

d) $\frac{4950}{5775}$

9. Completely factor each number into a product of primes.

a) $36$

b) $78$

c) $84$

d) $169$

e) $178$

f) $425$

g) $1000$

h) $6250$

i) $2431$

j) $5681$

10. Find the Greatest Common Factor of the following numbers.

a) 12, 28

b) 54, 66

c) 48, 136

d) 65, 169

e) 81, 108

f) 30, 45, 60

**Extending Level Questions**

11. Find the Lowest Common Multiple of the following numbers.

a) 7, 11, 13

b) 28, 35, 42

c) 22, 33, 66

d) 4, 36, 225

e) 8, 27, 125

f) 14, 84, 98

## Section 1.1a – Answer Key

1.
a) $50$
b) $0, 50$
c) $-12, 0, 50$
d) $-12, -2.7, 0, \frac{2}{3}, 4.21, 50$
e) $\pi$
f) $All$

2.
a) $12$
b) $0, 12$
c) $-4, 0, 12$
d) $-4, -0.3, 0, 2.3535 \ldots, 12$
e) $0.121121112 \ldots, \sqrt{10}$
f) $All$

3.
a) $\sqrt[3]{64}$
b) $\sqrt[3]{64}, \frac{0}{\sqrt{9}}$
c) $-\sqrt{64}, \sqrt[3]{64}, \frac{0}{\sqrt{9}}$
d) $All\ except\ \sqrt{0.4}$
e) $\sqrt{0.4}$
f) $All$

4.
a) $N, R, W, I, Real$
b) $I, Real$
c) $W, I, R, Real$
d) $R, Real$
e) $I, Real$
f) $R, Real$
g) $R, Real$
h) $R, Real$

5.
a) $1, 2, 3$
b) $6, 7, 8, \ldots$
c) $0, 1$
d) $-2, -1, 0, 1, \ldots$
e) $1, 2, 3, \ldots$
f) $\emptyset\ (Empty\ Set, None)$
g) $0, 1, 2, 3$
h) $-3, -2, -1, 0$

6.
a) $126, 280, 396, 2610, 7800$
b) $63, 126, 396, 2610, 7800$
c) $280, 396, 7800$
d) $280, 575, 2610, 7800$
e) $126, 396, 2610, 7800$
f) $63, 126, 396, 2610$
g) $280, 2610, 7800$

7.
a) $Prime$
b) $3 \cdot 17$
c) $3 \cdot 29$
d) $Prime$
e) $3 \cdot 3 \cdot 13$
f) $Prime$
g) $13 \cdot 47$
h) $Prime$
i) $17 \cdot 37$
j) $19 \cdot 29$

8.
a) $\frac{11}{13}$
b) $\frac{35}{36}$
c) $\frac{4}{5}$
d) $\frac{6}{7}$

9.
a) $2 \cdot 2 \cdot 3 \cdot 3$
b) $2 \cdot 3 \cdot 13$
c) $2 \cdot 2 \cdot 3 \cdot 7$
d) $13 \cdot 13$
e) $2 \cdot 89$
f) $5 \cdot 5 \cdot 17$
g) $2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$
h) $2 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$
i) $11 \cdot 13 \cdot 17$
j) $13 \cdot 19 \cdot 23$

10.
a) $4$
b) $6$
c) $8$
d) $13$
e) $27$
f) $15$

11.
a) $1001$
b) $420$
c) $66$
d) $900$
e) $27000$
f) $588$

**Extra Work Space**