Section 1.1b – Exponent Laws and Operations

This booklet belongs to:________________________________Block: _______

Remember How Brackets and Negatives Change the Question

- Where it gets tricky is with negative bases, and how the brackets, if any, are used.

Here we go…

$(-2)^2$ this means that everything inside the brackets is multiplied repeatedly

$$(-2)\cdot(-2)$$

- ➢ This has a profound effect on the final result

- A **negative number** multiplied an **even number** of times will always finish **POSITIVE**

So...

$$(-2)^4 = (-2)(-2)(-2)(-2)$$

$$= 4 \cdot 4$$

$$= 16$$

So, when we have an **EVEN POWER** we can **REWRITE** the statement without the brackets as a **POSTIVE statement**.

Watch this:

$$(-2)^4 = 2^4$$

This is a big deal

- A **negative number** multiplied an **odd number** of times will always finish **NEGATIVE**

$$(-2)^5 = (-2)(-2)(-2)(-2)(-2)$$

$$= 4 \cdot 4 \cdot (-2)$$

$$= 16 \cdot (-2)$$

$$-32$$

So, when we have an **ODD POWER** we can **REWRITE** the statement without the brackets as a **NEGATIVE statement**.

Watch this:

$$(-2)^5 = -2^5$$

This is a big deal

- Now we have covered when there are brackets
- But what about when there are no brackets?

So far, we know this…

$$(-a)^{Even} = a^{same\ power}$$

$$(-a)^{Odd} = -a^{same\ power}$$

But what does $-a$ mean? Let's look at it with a number.

Example:

$$-2 = (-1)2$$

So that means that…

$-2^3 = (-1)2^3$ also $-2^4 = (-1)2^4$

$= (-1) \cdot 2 \cdot 2 \cdot 2$ $= (-1) \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= -8$ $= -16$

Regardless of the power, even or odd, if there are **no brackets** to begin with the answer is **ALWAYS NEGATIVE**

Summary

- If the **negative is in brackets** then the result depends on the **exponents being odd or even**.

$(-2)^4 = 2^4$ Even exponent, the answer is always POSITIVE

$(-2)^5 = -2^5$ Odd exponent, the answer is always NEGATIVE

- If there are **NO BRACKETS**, the answer is **ALWAYS NEGATIVE**

$$-2^5 = (-1)2^5$$

$$-2^4 = (-1)2^4$$

Summary of the Exponents Laws from Grade 9

For any Integers m and n:		
Exponent of 1	$a^1 = a$	$3^1 = 3$
Exponent of 0	$a^0 = 1, \quad a \neq 0$	$(-5)^0 = 1$
Product Rule	$a^m \cdot a^n = a^{m+n}, \quad a \neq 0$	$2^3 \cdot 2^4 = 2^{3+4} = 2^7$
Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$	$\frac{3^5}{3^2} = 3^{5-2} = 3^3$
Power to a Power Rule	$(a^m)^n = a^{m \cdot n}$ $(ab)^n = a^n \cdot b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$(2^3)^4 = 2^{3 \cdot 4} = 2^{12}$ $(2x)^3 = 2^3 \cdot x^3 = 8x^3$ $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$
Negative Exponents	$a^{-2} = \frac{1}{a^2}$	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

Here are 3 New Concepts

Changing from Negative to Positive Exponents

Consider the expression: $\frac{1}{2^{-3}}$ By the negative exponent rule, this is the same as saying:

$$2^0 \div 2^{-3}$$

Which is equal to:

$$2^{0-(-3)} = 2^3$$

Changing from Negative to Positive Exponents

For any non-zero numbers $a\ and\ b$, with exponents $m\ and\ n$:

$$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m} \quad and \quad \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$

Example 1:

$$\left(\frac{2}{3}\right)^{-3} \to \left(\frac{3}{2}\right)^3 \to \frac{3^3}{2^3} \to \frac{27}{8}$$

$$\frac{3^{-2}}{4^{-3}} \to \frac{4^3}{3^2} \to \frac{64}{9}$$

Rational Exponent (Exponents that are FRACTIONS): $a^{\frac{1}{n}}$

Consider the square root example: $\sqrt{2} \cdot \sqrt{2} = 2$.

Now Consider the Exponent Rule example: $2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} = 2^1 = 2$

Since $\sqrt{2} \cdot \sqrt{2}$ and $2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}}$ equal 2, then $\sqrt{2} = 2^{\frac{1}{2}}$

Rational Exponents: $a^{\frac{1}{n}}$

For any non-negative real number a and any positive integer n:

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Example 2:

$$x^{\frac{1}{2}} = \sqrt{x}$$

$$\sqrt[4]{2} = 2^{\frac{1}{4}}$$

$$\sqrt{2} \cdot \sqrt[4]{2} = 2^{\frac{1}{2}} \cdot 2^{\frac{1}{4}} = 2^{\frac{1}{2}+\frac{1}{4}} = 2^{\frac{2}{4}+\frac{1}{4}} = 2^{\frac{3}{4}} = \sqrt[4]{2^3} = \sqrt[4]{8}$$

Rational Exponent (Exponents that are FRACTIONS): $a^{\frac{m}{n}}$

By the Power Rule: $8^{\frac{4}{3}} \rightarrow \left(8^{\frac{1}{3}}\right)^4 \rightarrow \left(\sqrt[3]{8}\right)^4 \rightarrow 2^4 \rightarrow 16$

We can write it this way too: $8^{\frac{4}{3}}$ can also be written as: $(8^4)^{\frac{1}{3}} \rightarrow \sqrt[3]{8^4} \rightarrow \sqrt[3]{4096} \rightarrow 16$

Decide which is easier and stick to that!

Rational Exponents: $a^{\frac{m}{n}}$

For any non-negative real number a and any positive integer n:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Example 3:

$$4^{\frac{3}{2}} \rightarrow (2^2)^{\frac{3}{2}} \rightarrow 2^{2\cdot\frac{3}{2}} \rightarrow 2^3 \rightarrow 8$$

$$\left(\sqrt[5]{32}\right)^4 \rightarrow 32^{\frac{4}{5}} \rightarrow (2^5)^{\frac{4}{5}} \rightarrow 2^4 \rightarrow 16$$

$$16^{-\frac{3}{4}} \rightarrow \frac{1}{16^{\frac{3}{4}}} \rightarrow \frac{1}{(2^4)^{\frac{3}{4}}} \rightarrow \frac{1}{2^3} \rightarrow \frac{1}{8}$$

$$\frac{\sqrt[3]{4}}{\sqrt[4]{2}} \rightarrow \frac{4^{\frac{1}{3}}}{2^{\frac{1}{4}}} \rightarrow \frac{(2^2)^{\frac{1}{3}}}{2^{\frac{1}{4}}} \rightarrow \frac{2^{\frac{2}{3}}}{2^{\frac{1}{4}}} \rightarrow 2^{\frac{2}{3}-\frac{1}{4}} \rightarrow 2^{\frac{8}{12}-\frac{3}{12}} \rightarrow \sqrt[12]{2^5} \rightarrow \sqrt[12]{32}$$

$$\sqrt[3]{9}\cdot\sqrt[4]{27} \rightarrow 9^{\frac{1}{3}}\cdot 27^{\frac{1}{4}} \rightarrow (3^2)^{\frac{1}{3}}\cdot(3^3)^{\frac{1}{4}} \rightarrow 3^{\frac{2}{3}}\cdot 3^{\frac{3}{4}} \rightarrow 3^{\frac{1}{4}+\frac{2}{3}} \rightarrow 3^{\frac{8}{12}+\frac{9}{12}} \rightarrow 3^{\frac{17}{12}}$$

This is good enough for now, but…

We'll learn how to go to this form in the next section

$$\sqrt[12]{3^{17}} \rightarrow \sqrt[12]{3^{12}\cdot 3^5} \rightarrow \mathbf{3\sqrt[12]{3^5}}$$

Section 1.1b – Practice Questions

Emerging Level Questions

1. Multiply. Leave answers in exponential form, positive exponents only

a) $2^3 \cdot 2^4$	b) $3^5 \cdot 3^7$
c) $4^{-3} \cdot 4^2$	d) $5^0 \cdot 5^3$
e) $a^2 \cdot a^3 \cdot a^{-5}$	f) $y^{-3} \cdot y^2 \cdot y$
g) $8^0 \cdot 8^1 \cdot 8^2$	h) $\left(\frac{2}{3}\right)^3 \cdot \left(\frac{2}{3}\right)^4$

i) $(-3)^4 \cdot (-3)^3 \cdot -3^2$

j) $\left(-\frac{1}{2}\right)^5 \cdot \left(-\frac{1}{2}\right)^{-3} \cdot \left(-\frac{1}{2}\right)^6$

2. Divide. Leave answers in exponential form, positive exponents only

a) $\frac{5^6}{5^3}$

b) $\frac{4^8}{4^4}$

c) $\frac{2^8}{2^2}$

d) $\frac{3^9}{3^3}$

e) $\frac{t^6}{t^2}$

f) $\frac{x^6}{x^6}$

g) $\frac{(-6)^4}{(-6)^{-3}}$

h) $\frac{(-9)^{-3}}{(-9)^{-6}}$

i) $\frac{(-2x)^3}{(-2x)^{-4}}$

j) $\frac{z^{-2}}{z^{-6}}$

3. Simplify. Express without brackets or negative exponents.

a) $(2^4)^2$

b) $(5^3)^{-2}$

c) $(3^{-4})^{-2}$

d) $(-3x^{-2})^0$

e) $(2x)^3$

f) $(3x^{-4})^2$

g) $(2a^{-4})^3$

h) $(3x^4y^{-2})^4$

i) $(-4a^{-3}b^{-2})^2$

j) $(-2^{-3}x^{-2}y)^3$

Proficient Level Questions

4. Simplify. Express without brackets or negative exponents.

a) $\frac{3^4 \cdot 3^7}{3^5}$

b) $\frac{2^5}{2^4 \cdot 2^3}$

c) $\frac{4^{-3} \cdot 4^1}{4^{-1}}$

d) $\frac{5^4 \cdot 5^{-2}}{5^3 \cdot 5^{-1}}$

e) $\frac{7^0 \cdot 7^{-3}}{7^2 \cdot 7^{-2}}$

f) $\frac{11^2 \cdot 11^3}{11^{-1}}$

g) $\frac{3(x^3)^2}{x^{-2}}$

h) $\frac{(3x^2)^{-3}}{x^3}$

i) $\frac{(2a^2b^{-4}c^{-5})^3}{2^2}$

j) $\left(\frac{2a^2}{3b^4}\right)^{-3}$

5. Solve.

a) 3^2

b) 3^{-2}

c) $\left(\frac{1}{3}\right)^2$

d) $\left(\frac{1}{3}\right)^{-2}$

e) -3^2

f) $(-3)^2$

g) $-\left(-\frac{1}{3}\right)^2$

h) $\left(-\frac{1}{3}\right)^2$

i) $\left(-\frac{1}{3}\right)^{-2}$

j) $-\left(-\frac{1}{3}\right)^{-2}$

k) -2^3

l) $-(-2)^3$

Extending Level Questions

6. Simplify. Express without brackets or negative exponents.

a) $\frac{(2a^2b^3)^{-2}(4ab^{-1})^3}{(a^3b)^{-4}}$

b) $\frac{(x^5y^2)^{-2}(x^2y^{-2})^3}{x^{-1}y^{-2}}$

c) $\dfrac{(5m^{-1}n^{2})^{2}(2m^{-2}n^{-3})^{3}}{(2m^{3}n^{2})^{-1}}$

d) $\dfrac{(3a^{-2}b^{3})^{2}(3a^{-1}b^{-4})^{-1}}{(3a^{2}b^{-2})^{-3}}$

e) $\dfrac{(3^{-1}x^{-2}y)^{-1}(5x^{2}y^{4})^{-2}}{(4x^{-2}y^{-3})^{2}}$

f) $\dfrac{(3^{-1}a^{-1}b^{-2})^{-2}(4a^{-3}b^{4})^{-2}}{(3a^{-3}b^{-4})^{2}}$

g) $\left(\frac{4^{-2}x^2y^{-3}}{x^{-2}y}\right)^3\left(\frac{8^{-1}x^{-3}y}{x^3y^{-1}}\right)^{-2}$

h) $\left(\frac{9ab^{-1}}{8a^{-2}b^2}\right)^{-2}\left(\frac{3a^{-2}b^2}{2a^2b^{-1}}\right)^3$

i) $\frac{(2x^{-1}y^2)(4x^2y^{-3})^{-2}}{(12x^2y^2)}$

j) $\left[\frac{(5x^{-3}y^4)^{-2}(6x^2y^{-5})}{15x^2y^{-4}}\right]^{-2}$

Proficient Level Questions

7. Solve.

a)	$16^{\frac{3}{4}}$	b)	$16^{-\frac{3}{4}}$
c)	$8^{\frac{2}{3}}$	d)	$8^{-\frac{2}{3}}$
e)	$27^{\frac{4}{3}}$	f)	$27^{-\frac{4}{3}}$
g)	$-16^{\frac{5}{4}}$	h)	$-16^{-\frac{5}{4}}$
i)	$-32^{\frac{4}{5}}$	j)	$-32^{-\frac{4}{5}}$
k)	$216^{\frac{2}{3}}$	l)	$216^{-\frac{2}{3}}$

m)	$-125^{\frac{4}{3}}$	n)	$-125^{-\frac{4}{3}}$
o)	$64^{\frac{7}{6}}$	p)	$64^{-\frac{7}{6}}$
q)	$-49^{\frac{3}{2}}$	r)	$-49^{-\frac{3}{2}}$
s)	$128^{\frac{5}{7}}$	t)	$128^{-\frac{5}{7}}$
u)	$-243^{\frac{6}{5}}$	v)	$-243^{-\frac{6}{5}}$
w)	$81^{\frac{5}{4}}$	x)	$81^{-\frac{5}{4}}$

8. Simplify. Leave answer with positive exponents.

a) $2^{\frac{1}{4}} \cdot 2^{\frac{5}{4}}$

b) $3^{\frac{2}{3}} \cdot 3^{\frac{7}{3}}$

c) $4^{\frac{1}{4}} \cdot 4^{-\frac{3}{4}}$

d) $5^{-\frac{2}{3}} \cdot 5^{-\frac{1}{3}}$

e) $\dfrac{6^{\frac{3}{4}}}{6^{\frac{5}{4}}}$

f) $\dfrac{7^{\frac{2}{5}}}{7^{-\frac{1}{5}}}$

g) $\dfrac{8^{-\frac{2}{7}} \cdot 8^{\frac{4}{7}}}{8^{-\frac{3}{7}}}$

h) $\dfrac{9^{\frac{3}{5}}}{9^{\frac{2}{5}} \cdot 9^{-\frac{4}{5}}}$

i) $a^{\frac{3}{4}} \cdot a^{\frac{5}{4}}$

j) $b^{\frac{5}{6}} \cdot b^{-\frac{1}{3}}$

k) $\frac{c^{\frac{2}{3}}}{c^{\frac{5}{6}}}$

l) $\frac{d^{\frac{1}{3}}}{d^{-\frac{1}{2}}}$

m) $\left(\frac{9}{4}\right)^{\frac{3}{2}}$

n) $\left(\frac{9}{4}\right)^{-\frac{3}{2}}$

o) $\left(\frac{81}{16}\right)^{\frac{3}{4}}$

p) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

q) $\left(a^3 b^{\frac{1}{4}}\right)^{\frac{2}{3}}$

r) $\left(x^4 y^{\frac{1}{2}}\right)^{\frac{4}{3}}$

s) $\left(a^{\frac{2}{3}} b^{\frac{5}{6}} c^{\frac{1}{2}}\right)^{\frac{6}{7}}$

t) $\left(x^{\frac{4}{3}} y^{\frac{3}{4}} z^{\frac{5}{2}}\right)^{-\frac{12}{5}}$

9. Simplify each radical. Assume the variables are positive.

a) $\sqrt[4]{4^1}$

b) $\sqrt[3]{8^6}$

c) $\sqrt[8]{16^3}$

d) $\sqrt[3]{27^2}$

e) $\sqrt[12]{9^3}$

f) $\sqrt[8]{4^2}$

g) $\sqrt[4]{a^2}$

h) $\sqrt[9]{b^3}$

10. Simplify.

a) $\sqrt{2} \cdot \sqrt[3]{2}$

b) $\sqrt{3} \cdot \sqrt[4]{3}$

c) $\sqrt[3]{2} \cdot \sqrt[4]{2}$

d) $\dfrac{\sqrt[3]{4}}{\sqrt[4]{4}}$

Extending Level Questions

e) $\dfrac{\sqrt{27}}{\sqrt[3]{9}}$

f) $\dfrac{\sqrt[3]{16}}{\sqrt[4]{8}}$

g) $\dfrac{\left(\frac{1}{2}\right)^x \cdot 8^x}{4^x}$

h) $\dfrac{3^x \cdot 27^x}{9^x}$

i) $\dfrac{\left(\frac{1}{3}\right)^{x} \cdot 81^{x}}{27^{x}}$

j) $\dfrac{5^{-x} \cdot 125^{2x}}{25^{3x}}$

Section 1.1b – Answer Key

1.
a) 2^7
b) 3^{12}
c) $4^{-1}\ or\ \frac{1}{4}$
d) 5^3
e) $a^0 = 1$
f) $y^0 = 1$
g) 8^3
h) $\left(\frac{2}{3}\right)^7$
i) 3^9
j) $\left(-\frac{1}{2}\right)^8$

2.
a) 5^3
b) 4^4
c) 2^6
d) 3^6
e) t^4
f) $x^0 = 1$
g) $(-6)^7$
h) $(-9)^3$
i) $(-2x)^7$
j) z^4

3.
a) 2^8
b) $5^{-6}\ or\ \frac{1}{5^6}$
c) 3^8
d) 1
e) $8x^3$
f) $\frac{9}{x^8}$
g) $\frac{8}{a^{12}}$
h) $\frac{81x^{16}}{y^8}$
i) $\frac{16}{a^6b^4}$
j) $-\frac{y^3}{512x^6}$

4.
a) 3^6
b) $\frac{1}{2^2}$
c) $\frac{1}{4}$
d) 1
e) $\frac{1}{7^3}$
f) 11^6
g) $3x^8$
h) $\frac{1}{27x^9}$
i) $\frac{2a^6}{b^{12}c^{15}}$
j) $\frac{27b^{12}}{8a^6}$

5.
a) 9
b) $\frac{1}{9}$
c) $\frac{1}{9}$
d) 9
e) -9
f) 9
g) $-\frac{1}{9}$
h) $\frac{1}{9}$
i) 9
j) -9
k) -8
l) 8

6.
a) $\frac{16a^{11}}{b^5}$
b) $\frac{1}{x^3y^8}$
c) $\frac{400}{m^5n^3}$
d) $81a^3b^4$
e) $\frac{3x^2}{400y^3}$
f) $\frac{a^{14}b^4}{16}$
g) $\frac{x^{24}}{64y^{16}}$
h) $\frac{8b^{15}}{3a^{18}}$
i) $\frac{y^6}{96x^7}$
j) $\frac{15\,625y^{18}}{4x^{12}}$

7.
a) 8
b) $\frac{1}{8}$
c) 4
d) $\frac{1}{4}$
e) 81
f) $\frac{1}{81}$
g) -32
h) $-\frac{1}{32}$
i) -16
j) $-\frac{1}{16}$
k) 36
l) $\frac{1}{36}$
m) -625
n) $-\frac{1}{625}$
o) 128
p) $\frac{1}{128}$
q) -343
r) $-\frac{1}{343}$
s) 32
t) $\frac{1}{32}$
u) -729
v) $-\frac{1}{729}$
w) 243
x) $\frac{1}{243}$

8.
a) $2^{\frac{3}{2}}$
b) 3^3
c) $\frac{1}{2}$
d) $\frac{1}{5}$
e) $\frac{1}{6^{\frac{1}{2}}}$
f) $7^{\frac{3}{5}}$
g) $2^{\frac{15}{7}}$
h) 9
i) a^2
j) $b^{\frac{1}{2}}$
k) $\frac{1}{c^{\frac{1}{6}}}$
l) $d^{\frac{5}{6}}$
m) $\frac{3^3}{2^3}$
n) $\frac{2^3}{3^3}$
o) $\frac{3^3}{2^3}$
p) $\frac{2^3}{3^3}$
q) $a^2b^{\frac{1}{6}}$
r) $x^{\frac{16}{3}}y^{\frac{2}{3}}$
s) $a^{\frac{4}{7}}b^{\frac{5}{7}}c^{\frac{3}{7}}$
t) $\frac{1}{x^{\frac{16}{5}}y^{\frac{9}{5}}z^6}$

9.
a) $\sqrt{2}$
b) 64
c) $2\sqrt{2}$
d) 9
e) $\sqrt{3}$
f) $\sqrt{2}$
g) $\sqrt{a}$
h) $\sqrt[3]{b}$

10.
a) $\sqrt[6]{32}$
b) $\sqrt[4]{27}$
c) $\sqrt[12]{128}$
d) $\sqrt[6]{2}$
e) $\sqrt[6]{243}$
f) $\sqrt[12]{128}$
g) 1
h) 9^x
i) 1
j) $\frac{1}{5^x}$

Extra Work Space