Section 1.2a – Perfect Squares, Cubes, and their Roots

This booklet belongs to:________________________________**Block:** _______

Squares and Square Roots

- To **square** a number is to raise the number to the **second** power
- A perfect Square then has **2 identical factors**

Example: $4^2 = 4 \cdot 4 = 16$
$9^2 = 9 \cdot 9 = 81$

- The identical factors are called the **square root** of a number
- The number with the rational square roots is called a **perfect square**
- We use the 'radical' or 'house' symbol $\sqrt{\ }$ to indicate square roots

Example 1: Determine which of the following are perfect squares.

a) 49 b) $\frac{4}{9}$

c) 7 d) $\frac{4}{15}$

Solution 1:

a) Yes, because $7 \cdot 7 = 49$, **two identical factors**

b) Yes, because $\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$, **two identical factors**

c) No, because 7 cannot be written as the product of two identical factors

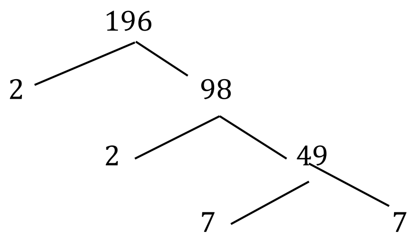
d) No, because $\frac{4}{15}$ cannot be written as the product of two identical factors

Determining Square Roots Without a Calculator

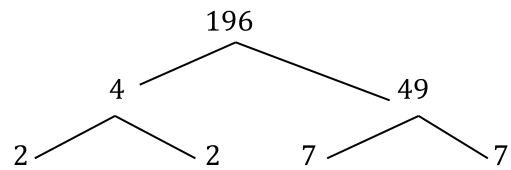
Using a Factor Tree

Example 2: Determine the square root of 196

Solution 2:



or

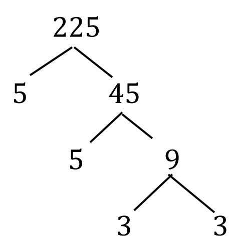


So, $\sqrt{196} = \sqrt{2 \cdot 2 \cdot 7 \cdot 7} = \sqrt{2 \cdot 2} \cdot \sqrt{7 \cdot 7} = 2 \cdot 7 = \mathbf{14}$

NOTE: For whole numbers $\sqrt{x^2} = \sqrt{x \cdot x} = \boldsymbol{x}$

Example 3: Determine the square root of 225

Solution 3:



So, $\sqrt{225} = \sqrt{3 \cdot 3 \cdot 5 \cdot 5} = \sqrt{3 \cdot 3} \cdot \sqrt{5 \cdot 5} = 3 \cdot 5 = \mathbf{15}$

Cubes and Cube Roots

- To cube a number is to raise the number to the **third** power

Example: $4^3 = 4 \cdot 4 \cdot 4 = 64$
$7^3 = 7 \cdot 7 \cdot 7 = 343$

- Some numbers can be written as the product of three identical factors
    - $27 = 3 \cdot 3 \cdot 3$
    - $125 = 5 \cdot 5 \cdot 5$

- The identical factors are called the **cube root** of a number
- The number with a rational cube root is called a **perfect cube**
- We use the 'radical' or 'house' symbol $\sqrt[3]{}$ to indicate cube roots (the little 3 is called the **index of the root**)

Example 4: Determine which are perfect cubes.

a) 8 b) $\frac{27}{64}$ c) 25 d) $\frac{8}{9}$

Solution 4:

a) Yes, because $2 \cdot 2 \cdot 2 = 8$, **three identical factors**

b) Yes, because $\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}$, **three identical factors**

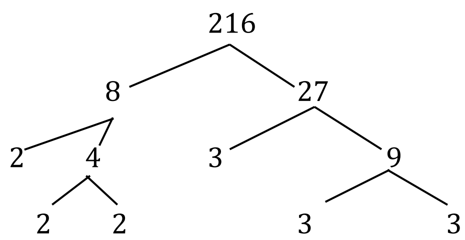
c) No, because 25 cannot be written as the product of three identical factors

d) No, because $\frac{8}{9}$ cannot be written as the product of three identical factors

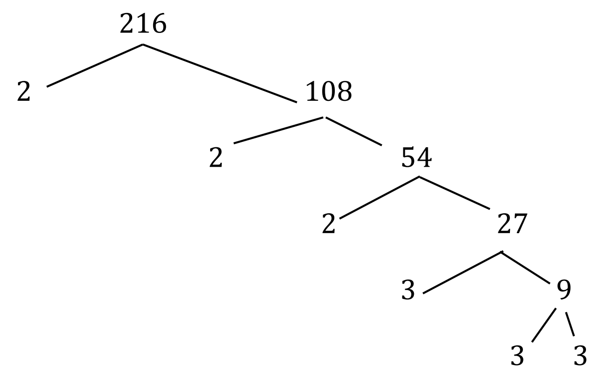
Determining Cube Roots Without a Calculator

Example 5: Determine the cube root of 216

Solution 5:



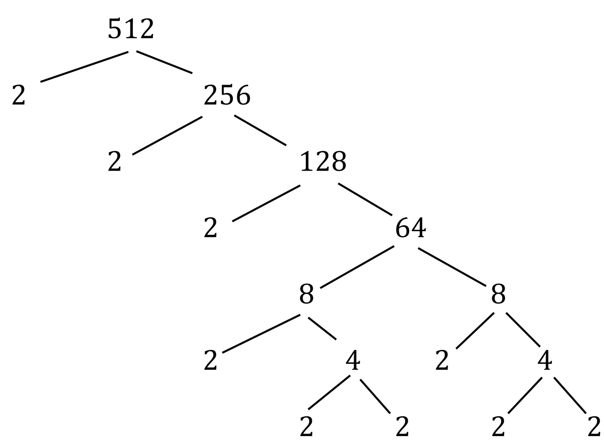
or



Therefore, $\sqrt[3]{216} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3} = \sqrt[3]{2 \cdot 2 \cdot 2} \cdot \sqrt[3]{3 \cdot 3 \cdot 3} = 2 \cdot 3 = \mathbf{6}$

Example 6: Determine the cube root of 512

Solution 6:



Therefore, $\sqrt[3]{512} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[3]{8 \cdot 8 \cdot 8} = 8$

Note: For whole numbers $\left(\sqrt[3]{x}\right)^3 = \left(\sqrt[3]{x^3}\right) = \sqrt[3]{x \cdot x \cdot x} = x$

Note: In the expression $\sqrt[k]{a}$, we call k the index, and assume $k \geq 2$. If the index is not written, the expression is assumed to be a square root, i.e. $k = 2$

Example: $\sqrt[5]{32} = 2$ because $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$, **five identical factors**

Section 1.2a – Practice Questions

Emerging Level Questions

1. Find the square root of the perfect squares without a calculator

a) $\sqrt{100}$

b) $\sqrt{441}$

c) $\sqrt{225}$

d) $\sqrt{361}$

e) $\sqrt{529}$

f) $\sqrt{2\,890\,000}$

2. Find the cube root of the perfect cubes without a calculator

a) $\sqrt[3]{27}$	b) $\sqrt[3]{1000}$
c) $\sqrt[3]{343}$	d) $\sqrt[3]{1728}$
e) $\sqrt[3]{3375}$	f) $\sqrt[3]{8000}$

PROFICIENT LEVEL QUESTIONS

3. Find the perfect square root, if it exists, without a calculator

a)	25	b)	29
c)	80	d)	81
e)	169	f)	99
g)	1600	h)	900

i)	$\frac{81}{400}$	j)	$\frac{8}{18}$

4. Find the perfect cube root, if it exists, without a calculator

a)	8	b)	9
c)	64	d)	81
e)	100	f)	216
g)	1000	h)	144

i) 625

j) 729

5. A cube has a volume of $216cm^3$. Determine the length of each side of the cube.

EXTENDING LEVEL QUESTIONS

6. The area of a rectangle with a length twice as long as the width is $1250m^2$. Determine the length and the width of the rectangle.

7. A rectangular solid has a length three times the width and a height twice its width. If the volume of the rectangle solid is $384in^3$, determine the dimensions of the rectangular solid.

Section 1.2a – Answer Key

1. a) 10 b) 21 c) 15 d) 19 e) 23 f) 1700
2. a) 3 b) 10 c) 7 d) 12 e) 15 f) 20
3. a) 5 b) Does Not Exist (DNE) c) DNE d) 9 e) 13 f) DNE g) 40 h) 30 i) $\frac{9}{20}$ j) $\frac{2}{3}$
4. a) 2 b) Does Not Exist (DNE) c) 4 d) DNE e) DNE f) 6 g) 10 h) DNE i) DNE j) 9
5. 6cm
6. $l = 50$m $w = 25$m
7. $l = 12$in $h = 8$in $w = 4$in

Extra Work Space