Section 1.2b – Irrational Numbers and Operations with Radicals

This booklet belongs to:________________________________Block:

- As we saw with perfect squares and perfect cubes, they have **rational roots**
- If a number is not rational then it must be irrational
- A **rational number** is one that can be represented as a **fraction**
- An **irrational number** is a **non-repeating, non-terminating decimal** value

Approximating Irrational numbers

- We can approximate a root by determining where the value lies on a number line

Example 1: Between what two consecutive integers is $\sqrt{7}$?

Solution 1:

- Let a and b be consecutive integers so that, $a < \sqrt{7} < b$.
- Therefore $a^2 < 7 < b^2$
- Since $2^2 = 4$ and $3^2 = 9$, then $2^2 < 7 < 3^2$
- Therefore, $2 < \sqrt{7} < 3$
- So $\sqrt{7}$ lies between 2 and 3

Example 2: Between what two consecutive integers is $\sqrt[3]{100}$?

Solution 2:

- Let a and b be consecutive integers so that, $a < \sqrt[3]{100} < b$.
- Therefore $a^3 < 100 < b^3$
- Since $4^3 = 64$ and $5^3 = 125$, then $4^3 < 100 < 5^3$
- Therefore, $4 < \sqrt[3]{100} < 5$
- So $\sqrt[3]{100}$ lies between 4 and 5

- Estimating is the mathematical skill often overlooked
- You need to be able to **think and understand** about the task at hand
- This will help you **avoid believing your calculator** when you've key something in wrong
- **Critical Thinking** and not just Procedural Thinking

Simplifying Radicals (Break out of the house)

- To factor a square root the product rule can be used.

> **The Product Rule for Square Roots**
>
> For any real numbers $\sqrt{A}$ and $\sqrt{B}$: $\sqrt{A \cdot B} = \sqrt{A} \cdot \sqrt{B}$

- The product rule is used when there is a **perfect square** as a factor.

Example 3: Consider $\sqrt{72}$. To simplify, there are many ways to factor 72

Solution 3:

Method 1: $\sqrt{72} = \sqrt{4 \cdot 18} \rightarrow \sqrt{4} \cdot \sqrt{18} \rightarrow 2 \cdot \sqrt{18} \rightarrow 2 \cdot \sqrt{9 \cdot 2}$

$= 2 \cdot \sqrt{9} \cdot \sqrt{2} \rightarrow 2 \cdot 3 \cdot \sqrt{2}$

$= 6\sqrt{2}$

Method 2: Quick and Easy. Look for the largest **perfect square factor** of 72.

$\sqrt{72} = \sqrt{36 \cdot 2} \rightarrow \sqrt{36} \cdot \sqrt{2} \rightarrow 6\sqrt{2}$

Example 4: Simplify $\sqrt{48}$

Solution 4:

$\sqrt{48} = \sqrt{3 \cdot 16} \rightarrow \sqrt{3} \cdot \sqrt{16} \rightarrow 4 \cdot \sqrt{3}$

- Radical expressions like $\sqrt{14}$ or $\sqrt{2}$ cannot be simplified, since neither have perfect square factors. Sometimes though, after multiplication, it is possible to simplify the product.

Example 5: $\sqrt{2} \cdot \sqrt{14} = \sqrt{28}$

$= \sqrt{4 \cdot 7} \rightarrow \sqrt{4} \cdot \sqrt{7}$

$= 2\sqrt{7}$

The Product Rule for Cube Roots

For any real number $\sqrt[3]{A}$ and $\sqrt[3]{B}$: $\sqrt[3]{A \cdot B} = \sqrt[3]{A} \cdot \sqrt[3]{B}$

- The product rule is used when there is a **perfect cube** as a factor.

Example 6: Simplify $\sqrt[3]{40}$

Solution 6: $\sqrt[3]{40} = \sqrt[3]{8 \cdot 5} \rightarrow \sqrt[3]{8} \cdot \sqrt[3]{5} \rightarrow 2\sqrt[3]{5}$

- Simplifying products of cube roots follows the method used for simplifying square roots

Example 7: Simplify $\sqrt[3]{9} \cdot \sqrt[3]{6}$

Solution 7:

$$\sqrt[3]{9} \cdot \sqrt[3]{6} = \sqrt[3]{54}$$

$$= \sqrt[3]{27 \cdot 2} \rightarrow \sqrt[3]{27} \cdot \sqrt[3]{2}$$

$$= 3\sqrt[3]{2}$$

Entire Radical

- An expression such as $2\sqrt{7}$ is called a **mixed** radical
- An expression such as $\sqrt{28}$ is called an **entire** radical
- Both expressions have the same value
- Any mixed radical can be changed to an entire radical

Example 8: Write the following as mixed radicals (non-simplified radicals) (put the number back in)

a) $9\sqrt{3}$ b) $2\sqrt[3]{5}$

Solution 8:

a) $9\sqrt{3} \rightarrow \sqrt{9 \cdot 9 \cdot 3} \rightarrow \sqrt{\mathbf{243}}$ b) $2\sqrt[3]{5} \rightarrow \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 5} \rightarrow \sqrt[3]{\mathbf{40}}$

Section 1.2b – Practice Questions

1. Without a calculator, decide if the numbers are rational or irrational.

Emerging Level Questions

a) $\sqrt{81}$

b) $\sqrt{810}$

c) $\sqrt{40}$

d) $\sqrt{400}$

Proficient Level Questions

e) $\sqrt{6.4}$

f) $\sqrt{0.64}$

g) $\sqrt{0.004}$

h) $\sqrt{0.0004}$

Emerging Level Questions

2. Without a calculator, decide if the numbers are rational or irrational.

a) $\sqrt[3]{1}$	b) $\sqrt[3]{10}$
c) $\sqrt[3]{100}$	d) $\sqrt[3]{1000}$
e) $\sqrt[3]{8}$	f) $\sqrt[3]{80}$

3. Without a calculator, approximate the irrational number to between two integers.

a) $\sqrt{30}$	b) $-\sqrt{58}$
c) $\sqrt{88}$	d) $-\sqrt{76}$

e) $\sqrt{130}$	f) $\sqrt[3]{98}$
g) $-\sqrt[3]{74}$	h) $\sqrt[3]{4}$

PROFICIENT LEVEL QUESTIONS

4. Find each product.

a) $\sqrt{3}\cdot\sqrt{5}$	b) $\sqrt{7}\cdot\sqrt{2}$	c) $\sqrt{13}\cdot\sqrt{13}$
d) $\sqrt{5}\cdot\sqrt{6}$	e) $\sqrt{2}\cdot\sqrt{3}\cdot\sqrt{5}$	f) $\sqrt[3]{4}\cdot\sqrt[3]{5}$

g) $\sqrt[3]{2} \cdot \sqrt[3]{2} \cdot \sqrt[3]{2}$

h) $\sqrt[3]{2} \cdot \sqrt[3]{3} \cdot \sqrt[3]{5}$

i) $\sqrt[3]{6} \cdot \sqrt[3]{7} \cdot \sqrt[3]{5}$

5. Which one of the radicals is simplified, explain how you know?

a) $\sqrt{44}, \sqrt{46}, \sqrt{48}, \sqrt{50}$

b) $\sqrt{18}, \sqrt{20}, \sqrt{21}, \sqrt{24}$

c) $\sqrt[3]{40}, \sqrt[3]{81}, \sqrt[3]{100}, \sqrt[3]{125}$

d) $\sqrt[3]{16}, \sqrt[3]{36}, \sqrt[3]{54}, \sqrt[3]{128}$

e) $\sqrt{32}, \sqrt[3]{32}, \sqrt{100}, \sqrt[3]{100}$

f) $\sqrt{64}, \sqrt[3]{64}, \sqrt{75}, \sqrt[3]{75}$

g) $\sqrt{27}, \sqrt[3]{27}, \sqrt{50}, \sqrt[3]{50}$

h) $\sqrt{8}, \sqrt{10}, \sqrt{12}, \sqrt{20}$

i) $\sqrt[3]{24}, \sqrt[3]{36}, \sqrt[3]{54}, \sqrt[3]{72}$

6. Simplify each radical.

a) $\sqrt{20}$	b) $\sqrt{72}$	c) $\sqrt{45}$
d) $\sqrt{24}$	e) $\sqrt{75}$	f) $\sqrt{125}$
g) $\sqrt{140}$	h) $\sqrt{128}$	i) $-\sqrt{80}$

7. Simplify each radical.

a) $2\sqrt{9}$	b) $4\sqrt{25}$	c) $6\sqrt{40}$
d) $3\sqrt{8}$	e) $4\sqrt{27}$	f) $6\sqrt{50}$
g) $-\frac{5}{2}\sqrt{32}$	h) $-2\frac{1}{3}\sqrt{72}$	i) $-\frac{4}{5}\sqrt{125}$

8. Simplify each radical.

a) $\sqrt[3]{40}$	b) $\sqrt[3]{48}$	c) $\sqrt[3]{54}$

d) $2\sqrt[3]{27}$

e) $-3\sqrt[3]{16}$

f) $\frac{1}{2}\sqrt[3]{64}$

9. Multiply, and simplify if possible.

a) $\sqrt{3} \cdot \sqrt{6}$

b) $\sqrt{7} \cdot \sqrt{14}$

c) $\sqrt{3} \cdot \sqrt{24}$

d) $5\sqrt{6} \cdot 2\sqrt{18}$

e) $-4\sqrt{10} \cdot \sqrt{21}$

f) $2\sqrt{10} \cdot 3\sqrt{50}$

g) $\sqrt[3]{4} \cdot \sqrt[3]{6}$

h) $2\sqrt[3]{12} \cdot \sqrt[3]{30}$

i) $-3\sqrt[3]{12} \cdot -2\sqrt[3]{18}$

10. Express as an entire radical.

a) $4\sqrt{3}$

b) $2\sqrt{5}$

c) $7\sqrt{6}$

d) $6\sqrt{3}$

e) $2\sqrt{5} \cdot \sqrt{3}$

f) $4\sqrt{5} \cdot 3\sqrt{3}$

11. Express as an entire radical.

a) $3\sqrt[3]{2}$	b) $4\sqrt[3]{3}$	c) $5\sqrt[3]{4}$
d) $7\sqrt[3]{8}$	e) $2\sqrt[3]{4}\cdot 5\sqrt[3]{5}$	f) $3\sqrt[3]{6}\cdot \sqrt[3]{7}$

Section 1.2b – Answer Key

1.
a) R
b) I
c) I
d) R
e) I
f) R
g) I
h) R

2.
a) R
b) I
c) I
d) R
e) R
f) I

3.
a) $5 < x < 6$
b) $-8 < x < -7$
c) $9 < x < 10$
d) $-9 < x < -8$
e) $11 < x < 12$
f) $4 < x < 5$
g) $-5 < x < -4$
h) $1 < x < 2$

4.
a) $\sqrt{15}$
b) $\sqrt{14}$
c) 13
d) $\sqrt{30}$
e) $\sqrt{30}$
f) $\sqrt[3]{20}$
g) 2
h) $\sqrt[3]{30}$
i) $\sqrt[3]{210}$

5.
a) $\sqrt{46}$
b) $\sqrt{21}$
c) $\sqrt[3]{100}$
d) $\sqrt[3]{36}$
e) $\sqrt[3]{100}$
f) $\sqrt[3]{75}$
g) $\sqrt[3]{50}$
h) $\sqrt{10}$
i) $\sqrt[3]{36}$

6.
a) $2\sqrt{5}$
b) $6\sqrt{2}$
c) $3\sqrt{5}$
d) $2\sqrt{6}$
e) $5\sqrt{3}$
f) $5\sqrt{5}$
g) $2\sqrt{35}$
h) $8\sqrt{2}$
i) $-4\sqrt{5}$

7.
a) 6
b) 20
c) $12\sqrt{10}$
d) $6\sqrt{2}$
e) $12\sqrt{3}$
f) $30\sqrt{2}$
g) $-10\sqrt{2}$
h) $-14\sqrt{2}$
i) $-4\sqrt{5}$

8.
a) $2\sqrt[3]{5}$
b) $2\sqrt[3]{6}$
c) $3\sqrt[3]{2}$
d) 6
e) $-6\sqrt[3]{2}$
f) 2

9.
a) $3\sqrt{2}$
b) $7\sqrt{2}$
c) $6\sqrt{2}$
d) $60\sqrt{3}$
e) $-4\sqrt{210}$
f) $60\sqrt{5}$
g) $2\sqrt[3]{3}$
h) $4\sqrt[3]{45}$
i) 36

10.
a) $\sqrt{48}$
b) $\sqrt{20}$
c) $\sqrt{294}$
d) $\sqrt{108}$
e) $\sqrt{60}$
f) $\sqrt{2160}$

11.
a) $\sqrt[3]{54}$
b) $\sqrt[3]{192}$
c) $\sqrt[3]{500}$
d) $\sqrt[3]{2744}$
e) $\sqrt[3]{20\,000}$
f) $\sqrt[3]{1134}$

Extra Work Space