Section 1.2b – Practice Questions

1. Without a calculator, decide if the numbers are rational or irrational.

Emerging Level Questions

a) $\sqrt{81}$

81 → 9, 9

$\sqrt{81} = 9 = \boxed{\text{rational}}$

b) $\sqrt{810}$

810 → 405, 2; 405 → 81, 5; 81 → 9, 9

$\underbrace{\text{factors: } 9 \cdot 9 \cdot 5 \cdot 2}$

no identical two factors so $\boxed{\text{Irrational}}$

c) $\sqrt{40}$

40 → 20, 2; 20 → 10, 2; 10 → 2, 5

$\underbrace{\text{factors: } 5 \cdot 2 \cdot 2 \cdot 2}$

no two identical factors so $\boxed{I}$

d) $\sqrt{400}$

400 → 200, 2; 200 → 100, 2; 100 → 50, 2; 50 → 25, 2; 25 → 5, 5

$\underbrace{\text{factors: } 5 \cdot 5 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$

Identical factors of $\underline{20}$

$\sqrt{400} = 20 = \boxed{R}$

Proficient Level Questions

e) $\sqrt{6.4}$

$6.4 = \frac{64}{10}$

$\sqrt{\frac{64}{10}} = \frac{\sqrt{64}}{\sqrt{10}}$

$\frac{8}{\sqrt{10}}$ ← irrational baby!

f) $\sqrt{0.64}$

$0.64 = \frac{64}{100}$

$\sqrt{\frac{64}{100}} = \frac{\sqrt{64}}{\sqrt{100}} = \frac{8}{10} = \boxed{R}$

g) $\sqrt{0.004}$

$0.004 = \frac{4}{1000}$

$\sqrt{\frac{4}{1000}} = \frac{\sqrt{4}}{\sqrt{1000}} = \frac{2}{\sqrt{1000}} = \boxed{I}$

h) $\sqrt{0.0004}$

$\sqrt{0.0004} = \sqrt{\frac{4}{10000}} = \frac{\sqrt{4}}{\sqrt{10000}} = \frac{2}{100} = \boxed{R}$

Emerging Level Questions

2. Without a calculator, decide if the numbers are rational or irrational.

a) $\sqrt[3]{1}$

$\underbrace{\text{factors: } 1\cdot1\cdot1\cdots}$

Three identical factors so

$\sqrt[3]{1} = 1 = \boxed{R}$

b) $\sqrt[3]{10}$

10 → 5, 2

$\underbrace{\text{factors: } 5\cdot2}$

No three identical factors so $\boxed{I}$

c) $\sqrt[3]{100}$

100 → 10, 10; 10 → 2, 5; 10 → 2, 5

$\underbrace{\text{factors: } 2\cdot2\cdot5\cdot5}$

No three identical factors so

$\sqrt[3]{100} = \boxed{I}$

d) $\sqrt[3]{1000}$

Three identical factors so

1000 → 100, 10; 100 → 10, 10

$\sqrt[3]{1000} = 10 = \boxed{R}$

e) $\sqrt[3]{8}$

Three identical factors so

8 → 4, 2; 4 → 2, 2

$\sqrt[3]{8} = 2 = \boxed{R}$

f) $\sqrt[3]{80}$

80 → 40, 2; 40 → 20, 2; 20 → 5, 2

$\underbrace{\text{factors: } 2\cdot2\cdot2\cdot5}$

No identical factors so $\sqrt[3]{80} = \boxed{I}$

3. Without a calculator, approximate the irrational number to between two integers.

a) $\sqrt{30}$

$\boxed{5 < \sqrt{30} < 6}$

$\sqrt{1} = 1$
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\leftarrow \sqrt{25} = 5$
$\leftarrow \sqrt{36} = 6$

b) $-\sqrt{58}$

$(-1)\sqrt{49} = -7$
$(-1)\sqrt{64} = -8$

$\boxed{-8 < -\sqrt{58} < -7}$

-8, $-\sqrt{58}$, -7, 0

c) $\sqrt{88}$

$\sqrt{64} = 8$
$\leftarrow \sqrt{81} = 9$
$\leftarrow \sqrt{100} = 10$

$\boxed{9 < \sqrt{88} < 10}$

d) $-\sqrt{76}$

$(-1)(\sqrt{64}) = -8$
$(-1)(\sqrt{81}) = -9$

$\boxed{-9 < -\sqrt{76} < -8}$

-9, $-\sqrt{76}$, -8, 0

e) $\sqrt{130}$

$\sqrt{121} = 11$
$\sqrt{144} = 12$

$\boxed{11 < \sqrt{130} < 12}$

f) $\sqrt[3]{98}$

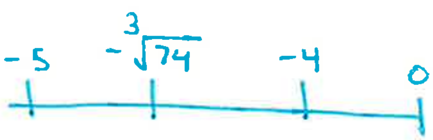
$\sqrt[3]{64} = 4$
$\sqrt[3]{125} = 5$

$\boxed{4 < \sqrt[3]{98} < 5}$

g) $-\sqrt[3]{74}$

$(-1)\sqrt[3]{64} = -4$
$(-1)\sqrt[3]{125} = -5$

$\boxed{-5 < -\sqrt[3]{74} < -4}$



h) $\sqrt[3]{4}$

$\sqrt[3]{1} = 1$
$\sqrt[3]{8} = 2$

$\boxed{1 < \sqrt[3]{4} < 2}$

PROFICIENT LEVEL QUESTIONS

4. Find each product.

a) $\sqrt{3} \cdot \sqrt{5}$

$\sqrt{3} \cdot \sqrt{5} = \sqrt{3 \cdot 5} = \boxed{\sqrt{15}}$

b) $\sqrt{7} \cdot \sqrt{2}$

$= \sqrt{7 \cdot 2} = \boxed{\sqrt{14}}$

c) $\sqrt{13} \cdot \sqrt{13}$

$= \underbrace{\sqrt{13 \cdot 13} = \sqrt{169}}$

Two identical factors so

$= \boxed{13}$

d) $\sqrt{5} \cdot \sqrt{6}$

$= \sqrt{5 \cdot 6} = \boxed{\sqrt{30}}$

e) $\sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5}$

$= \sqrt{2 \cdot 3 \cdot 5} = \boxed{\sqrt{30}}$

f) $\sqrt[3]{4} \cdot \sqrt[3]{5}$

same so ✓

$\sqrt{4 \cdot 5} = \boxed{\sqrt{20}}$

g) $\sqrt[3]{2} \cdot \sqrt[3]{2} \cdot \sqrt[3]{2}$

$\underbrace{= \sqrt[3]{2 \cdot 2 \cdot 2} = \sqrt[3]{8}}$

Three identical factors so

$\sqrt[3]{8} = \boxed{2}$

h) $\sqrt[3]{2} \cdot \sqrt[3]{3} \cdot \sqrt[3]{5}$

$\sqrt{2 \cdot 3 \cdot 5} = \boxed{\sqrt[3]{30}}$

i) $\sqrt[3]{6} \cdot \sqrt[3]{7} \cdot \sqrt[3]{5}$

$\sqrt[3]{6 \cdot 7 \cdot 5} = \boxed{\sqrt[3]{210}}$

5. Which one of the radicals is simplified, explain how you know? Looking for perfect square/cube factors

a) $\sqrt{44}, \sqrt{46}, \sqrt{48}, \sqrt{50}$

$4 \cdot 11$ — Simplify

No perfect square factors — $2 \cdot 23$ simplified

$16 \cdot 3$

$25 \cdot 2$

b) $\sqrt{18}, \sqrt{20}, \sqrt{21}, \sqrt{24}$

$2 \cdot 9$

$4 \cdot 5$

Simplified $3 \cdot 7$

$4 \cdot 6$

c) $\sqrt[3]{40}, \sqrt[3]{81}, \sqrt[3]{100}, \sqrt[3]{125}$

$8 \cdot 5$ — Perfect cube

$27 \cdot 3$ — PC

Simplified

$5 \cdot 5 \cdot 5$

d) $\sqrt[3]{16}, \sqrt[3]{36}, \sqrt[3]{54}, \sqrt[3]{128}$

$8 \cdot 2$

Simplified

$27 \cdot 2$

128
2 64
2 32
2 16

e) $\sqrt{32}, \sqrt[3]{32}, \sqrt{100}, \sqrt[3]{100}$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$2 \cdot 2 \cdot 2$
$2 \cdot 2$

$10 \cdot 10$

$10 \cdot 10$ Simplified

f) $\sqrt{64}, \sqrt[3]{64}, \sqrt{75}, \sqrt[3]{75}$

$8 \cdot 8$

$8 \cdot 8$

$5 \cdot 5 \cdot 3$

$5 \cdot 5 \cdot 3$ Simplified

g) $\sqrt{27}, \sqrt[3]{27}, \sqrt{50}, \sqrt[3]{50}$

$3 \cdot 3 \cdot 3$ perfect cube. $5 \cdot 5 \cdot 2$ simplified

h) $\sqrt{8}, \sqrt{10}, \sqrt{12}, \sqrt{20}$

$2 \cdot 2 \cdot 2$ $2 \cdot 5$ simplified $2 \cdot 2 \cdot 3$ $2 \cdot 5 \cdot 2$

i) $\sqrt[3]{24}, \sqrt[3]{36}, \sqrt[3]{54}, \sqrt[3]{72}$

$2 \cdot 2 \cdot 2 \cdot 6$ $3 \cdot 3 \cdot 2 \cdot 2$ simplified $2 \cdot 3 \cdot 3 \cdot 3$ $2 \cdot 3 \cdot 3 \cdot 2 \cdot 2$

6. Simplify each radical.

a) $\sqrt{20}$

factors: $5 \cdot 2 \cdot 2$

$2\sqrt{5}$

20 → 10, 2 → 5, 2

b) $\sqrt{72}$

factors: $3 \cdot 3 \cdot 2 \cdot 2 \cdot 2$

$= 3 \cdot 2 \cdot \sqrt{2}$

$= \boxed{6\sqrt{2}}$

72 → 36, 2 → 18, 2 → 9, 2 → 3, 3

c) $\sqrt{45}$

factors: $3 \cdot 3 \cdot 5$

$= \boxed{3\sqrt{5}}$

45 → 9, 5 → 3, 3

d) $\sqrt{24}$

factors: $2 \cdot 2 \cdot 2 \cdot 3$

$= 2\sqrt{2 \cdot 3}$

$= \boxed{2\sqrt{6}}$

24 → 12, 2 → 6, 2 → 2, 3

e) $\sqrt{75}$

factors: $5 \cdot 5 \cdot 3$

$\boxed{5\sqrt{3}}$

75 → 15, 5 → 3, 5

f) $\sqrt{125}$

factors: $5 \cdot 5 \cdot 5$

$\boxed{5\sqrt{5}}$

125 → 25, 5 → 5, 5

g) $\sqrt{140}$

factors: $2 \cdot 2 \cdot 5 \cdot 7$

$= 2\sqrt{5 \cdot 7}$

$= \boxed{2\sqrt{35}}$

140 → 70, 2 → 10, 7 → 2, 5

h) $\sqrt{128}$

factors: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$2 \cdot 2 \cdot 2\sqrt{2}$

$\boxed{8\sqrt{2}}$

128 → 64, 2 → 32, 2 → 16, 2 → 8, 2 → 4, 2 → 2, 2

i) $-\sqrt{80}$

factors: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$

$(-1)(2)(2)\sqrt{5}$

$\boxed{-4\sqrt{5}}$

80 → 40, 2 → 20, 2 → 10, 2 → 5, 2

7. Simplify each radical.

a) $2\sqrt{9}$ factors: $3\cdot3$ $= 2(3)$ $= \boxed{6}$	b) $4\sqrt{25}$ factors: $5\cdot5$ $= 4(5)$ $= \boxed{20}$	c) $6\sqrt{40}$ factors: $2\cdot2\cdot2\cdot5$ $6(2)\sqrt{2\cdot5}$ $\boxed{12\sqrt{10}}$
d) $3\sqrt{8}$ factors: $2\cdot2\cdot2$ $= 3(2)\sqrt{2}$ $= \boxed{6\sqrt{2}}$	e) $4\sqrt{27}$ factors: $3\cdot3\cdot3$ $= 4(3)\sqrt{3}$ $= \boxed{12\sqrt{3}}$	f) $6\sqrt{50}$ factors: $5\cdot5\cdot2$ $6(5)\sqrt{2}$ $\boxed{30\sqrt{2}}$
g) $-\frac{5}{2}\sqrt{32}$ factors: $2\cdot2\cdot2\cdot2\cdot2$ $\frac{(-1)5\cdot2\cdot\cancel{2}}{\cancel{2}}\sqrt{2}$ $\boxed{-10\sqrt{2}}$	h) $-2\frac{1}{3}\sqrt{72}$ factors: $2\cdot2\cdot2\cdot3\cdot3$ $= -\frac{7}{\cancel{3}}\cdot\cancel{6}\sqrt{2}$ $= \boxed{-14\sqrt{2}}$	i) $-\frac{4}{5}\sqrt{125}$ factors: $5\cdot5\cdot5$ $= -\frac{4}{\cancel{5}}(\cancel{5})(\sqrt{5})$ $= \boxed{-4\sqrt{5}}$

8. Simplify each radical. (Need 3 identical factors)

a) $\sqrt[3]{40}$ $\boxed{2\sqrt[3]{5}}$	b) $\sqrt[3]{48}$ $\boxed{2\sqrt[3]{6}}$ Perfect cube	c) $\sqrt[3]{54}$ $\boxed{3\sqrt[3]{2}}$

d) $2\sqrt[3]{27}$

$27 \to 9 \cdot 3 \to 3 \cdot 3 \cdot 3$

$2 \cdot 3$

$= \boxed{6}$

e) $-3\sqrt[3]{16}$

$16 \to 8 \cdot 2 \to 4 \cdot 2 \to 2 \cdot 2$

$-3 \cdot 2 \cdot \sqrt[3]{2}$

$\boxed{-6\sqrt[3]{2}}$

f) $\frac{1}{2}\sqrt[3]{64}$

$64 \to 32 \cdot 2 \to 16 \cdot 2 \to 8 \cdot 2 \to 4 \cdot 2 \to 2 \cdot 2$

$\cancel{\frac{1}{2}} \cdot \cancel{2} \cdot 2$

$\boxed{2}$

9. **Multiply, and simplify if possible.**

a) $\sqrt{3} \cdot \sqrt{6}$

$\sqrt{3 \cdot 6}$

$\sqrt{18}$

$18 \to 6 \cdot 3 \to 3 \cdot 2$

$\boxed{3\sqrt{2}}$

b) $\sqrt{7} \cdot \sqrt{14}$

$\sqrt{7 \cdot 14}$

$\sqrt{98}$

$98 \to 14 \cdot 7 \to 7 \cdot 2$

$\boxed{7\sqrt{2}}$

c) $\sqrt{3} \cdot \sqrt{24}$

$\sqrt{72}$

$72 \to 24 \cdot 3 \to 8 \cdot 3 \to 4 \cdot 2 \to 2 \cdot 2$

$3 \cdot 2\sqrt{2}$

$\boxed{6\sqrt{2}}$

d) $5\sqrt{6} \cdot 2\sqrt{18}$

$5 \cdot 2 \cdot \sqrt{6 \cdot 18}$

$5 \cdot 2 \cdot \sqrt{108}$

$108 \to 54 \cdot 2 \to 27 \cdot 2 \to 9 \cdot 3 \to 3 \cdot 3$

$5 \cdot 2 \cdot 2 \cdot 3\sqrt{3}$

$\boxed{60\sqrt{3}}$

e) $-4\sqrt{10} \cdot \sqrt{21}$

$-4\sqrt{10 \cdot 21}$

$\boxed{-4\sqrt{210}}$

$210 \to 105 \cdot 2 \to 21 \cdot 5 \to 7 \cdot 3$

$\underbrace{7 \quad 3}$

no same two

f) $2\sqrt{10} \cdot 3\sqrt{50}$

$2 \cdot 3\sqrt{500}$

$500 \to 100 \cdot 5 \to 50 \cdot 2 \to 10 \cdot 5 \to 5 \cdot 2$

$2 \cdot 3 \cdot 2 \cdot 5\sqrt{5}$

$\boxed{60\sqrt{5}}$

g) $\sqrt[3]{4} \cdot \sqrt[3]{6}$

$\sqrt[3]{24}$

$\boxed{2\sqrt[3]{3}}$

24, 12 (2), 6 (2), 3 (2)

h) $2\sqrt[3]{12} \cdot \sqrt[3]{30}$

$2\sqrt[3]{360}$

360, 180 (2), 90 (2), 45 (2), 9 5, 3 3

$2 \cdot 2\sqrt[3]{15}$

$\boxed{4\sqrt[3]{15}}$

i) $-3\sqrt[3]{12} \cdot -2\sqrt[3]{18}$

$-3 \cdot -2\sqrt[3]{216}$

$\cancel{+}3 \cdot \cancel{/}2 \cdot 3 \cdot 2$

$\boxed{36}$

216, 108 (2), 54 (2), 27 (2), 9 (3), (3)(3)

10. Express as an entire radical.

a) $4\sqrt{3}$

$4 = \sqrt{4 \cdot 4}$

$\sqrt{4 \cdot 4} \cdot \sqrt{3}$

$\boxed{\sqrt{48}}$

b) $2\sqrt{5}$

$2 = \sqrt{4}$

$\sqrt{4} \cdot \sqrt{5}$

$\boxed{\sqrt{20}}$

c) $7\sqrt{6}$

$7 = \sqrt{49}$

$\sqrt{49} \cdot \sqrt{6}$

$\boxed{\sqrt{294}}$

d) $6\sqrt{3}$

$6 = \sqrt{36}$

$\sqrt{36} \cdot \sqrt{3}$

$\boxed{\sqrt{108}}$

e) $2\sqrt{5} \cdot \sqrt{3}$

$\sqrt{4} = 2$

$\sqrt{4} \cdot \sqrt{5} \cdot \sqrt{3}$

$\boxed{\sqrt{60}}$

f) $4\sqrt{5} \cdot 3\sqrt{3}$

$4 = \sqrt{16}$

$3 = \sqrt{9}$

$\sqrt{16} \cdot \sqrt{5} \cdot \sqrt{3} \cdot \sqrt{9}$

$\boxed{\sqrt{2160}}$

11. Express as an entire radical.

a) $3\sqrt[3]{2}$

$3 = \sqrt[3]{27}$

$\sqrt[3]{27} \cdot \sqrt[3]{2}$

$\boxed{\sqrt[3]{54}}$

b) $4\sqrt[3]{3}$

$4 = \sqrt[3]{64}$

$\sqrt[3]{64} \cdot \sqrt[3]{3}$

$\boxed{\sqrt[3]{192}}$

c) $5\sqrt[3]{4}$

$5 = \sqrt[3]{125}$

$\sqrt[3]{125} \cdot \sqrt[3]{4}$

$\boxed{\sqrt[3]{500}}$

d) $7\sqrt[3]{8}$

$7 = \sqrt[3]{343}$

$\sqrt[3]{343} \cdot \sqrt[3]{8}$

$\boxed{\sqrt[3]{2744}}$

e) $2\sqrt[3]{4} \cdot 5\sqrt[3]{5}$

$2 = \sqrt[3]{8}$

$5 = \sqrt[3]{125}$

$\sqrt[3]{8} \cdot \sqrt[3]{4} \cdot \sqrt[3]{125} \cdot \sqrt[3]{5}$

$\boxed{\sqrt[3]{20000}}$

f) $3\sqrt[3]{6} \cdot \sqrt[3]{7}$

$3 = \sqrt[3]{27}$

$\sqrt[3]{27} \cdot \sqrt[3]{6} \cdot \sqrt[3]{7}$

$\boxed{\sqrt[3]{1134}}$