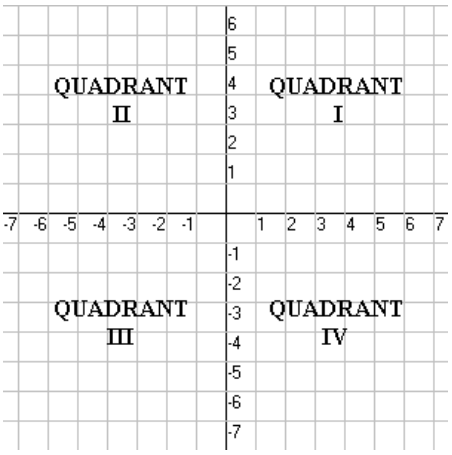
Section 2.1a – Relations and Functions

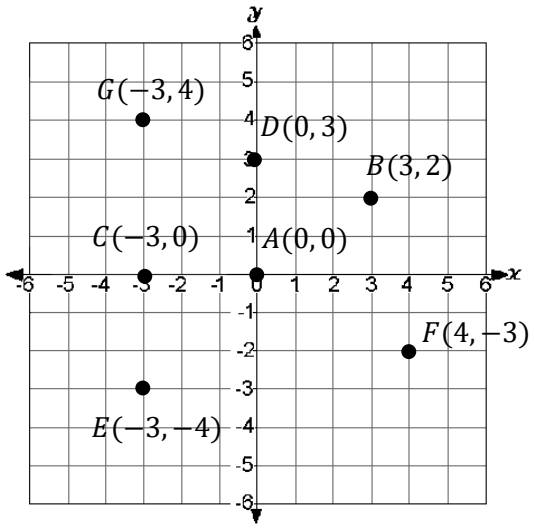
This booklet belongs to:_____Block: _____

- We use **graphs** in Math, like History books use pictures. Graphs give us a quick way to make comparisons, draw conclusions, and approximate quantities. The next section will involve different names of graphing relationships and how to plot and read information.

Coordinate System

- Similar to a real number on a real number line, **ordered pairs** can be represented by points on the Cartesian Plane.
- Ordered pairs are written in the form (x, y)
- There is a unique point on the plane that corresponds to every ordered pair





- The ordered pair $\boldsymbol{A}$ $(\mathbf{0}, \mathbf{0})$ is located **at the origin**.
- The ordered pair $\boldsymbol{B}$ $(\mathbf{3}, \mathbf{2})$ is located **three units to the right and two units up** from the origin.
- The ordered pair $\boldsymbol{C}$ $(-\mathbf{3}, \mathbf{0})$ is located **three units to the left** of the origin on the $\boldsymbol{x - axis}$**.**
- The ordered pair $\boldsymbol{D}$ $(\mathbf{0}, \mathbf{3})$ is located **three units up** from the origin on the $\boldsymbol{y - axis}$**.**
- The ordered pair $\boldsymbol{E}$ $(-\mathbf{3}, -\mathbf{4})$ is located **three units to the left** and **four units down** from the origin.
- The ordered pair $\boldsymbol{F}$ $(\mathbf{4}, -\mathbf{3})$ is located **four units to the right** and **three units down** from the origin.
- The ordered pair $\boldsymbol{G}$ $(-\mathbf{3}, \mathbf{4})$ is located **three units to the left** and **four units up** from the origin.

Ordered pairs $(\mathbf{4}, -\mathbf{3})$ $\boldsymbol{and}$ $(-\mathbf{3}, \mathbf{4})$ plot different points. That is why they are called **ordered pairs**, it makes a **significant difference** which **number comes first**.

Relations

- Relations are sets of **ordered pairs** (x, y)
- The set of the first components, or $\boldsymbol{x - values}$, is the **DOMAIN**
- The set of the second components or $\boldsymbol{y - values}$, is called the **RANGE**
- To find solutions to a relation, values are arbitrarily assigned for the $\boldsymbol{x\ term}$ from the set of real numbers.
- This makes $\boldsymbol{x}$ the $\boldsymbol{independent\ variable}$
- Choosing input values for x provides us with output values for y
- This makes $\boldsymbol{y}$ the $\boldsymbol{dependant\ variable}$

Example:

$Input$ x	$Relation$ $y = 2x + 1$	$Output$ y
-3	$2(-3) + 1$	-5
0	$2(0) + 1$	1
2	$2(2) + 1$	5

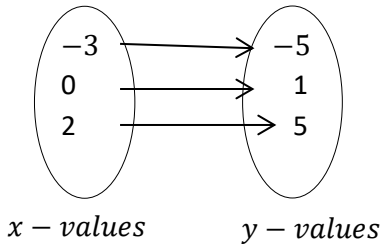
These values represent **3 solutions** to the **infinitely many** for the relation $y = 2x + 1$.

These solutions can be represented as:

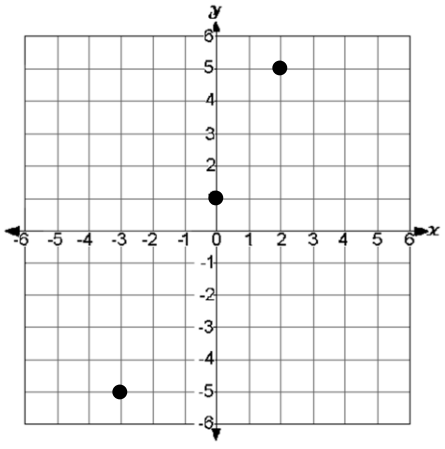
1. Ordered pairs: $(-3, -5)$, $(0, 1)$, $(2, 5)$
2. In a table:

x	-3	0	2
y	-5	1	5

3. Using Mapping Notation



4. Or by graphing



Functions

- A function is a special type of relation

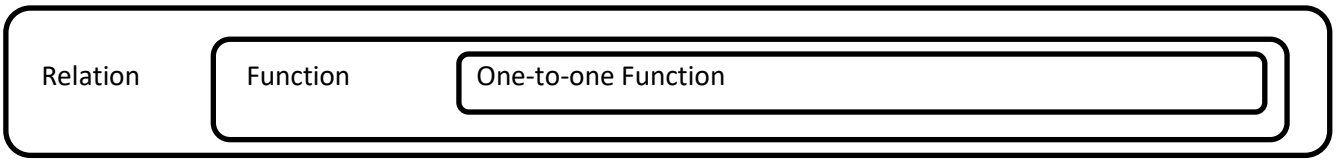
Function

- For every value of the $\boldsymbol{domain}$ $(\boldsymbol{x-value})$, there is **one and only one**, value for the $\boldsymbol{range}$ $(\boldsymbol{y-value})$
- Each element in the **domain** corresponds to **exactly one** element in the **range**.

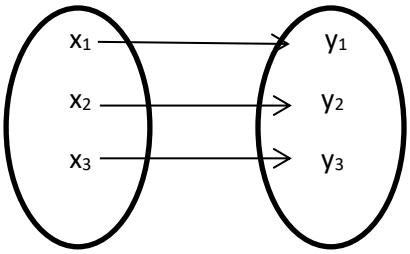
One-to-One Function

- A function in which **every individual value** of the $\boldsymbol{domain}$ $(\boldsymbol{x-value})$ is associated with **one value** of the $\boldsymbol{range}$ $(\boldsymbol{y-value})$, and vice versa.
- This means that if the function is a **one-to-one function**, then for each x in the domain, there is **one, and only one**, y in the range, and no y in the range is the image **of more than one** x in the domain.

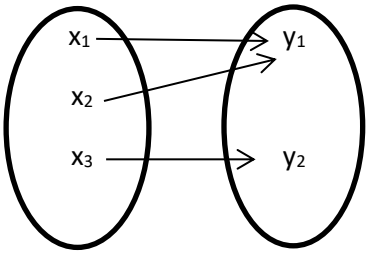
Hierarchy of Relations, Functions, and One-to-one Functions



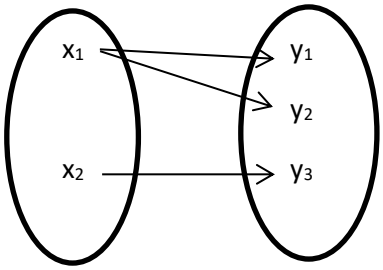
Example:



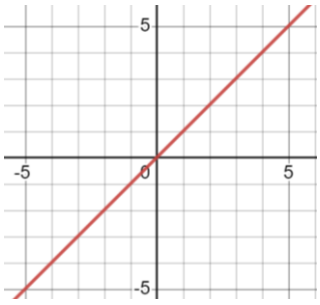
1-1

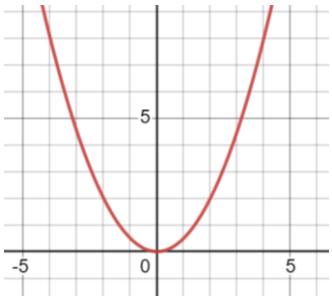


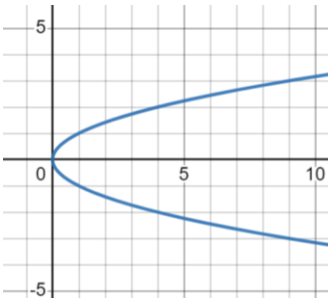
A function, not 1-1



Not a function, just a Relation







Note: The **Range (Output)** depends on the **Domain (Input)**

Example 3: Given the ordered pairs: $(-5, 4), (-3, 2), (-2, 0), (0, -2), (1, -3), (4, -4)$, what is the value of $\boldsymbol{y}$ $(\boldsymbol{output})$ when $\boldsymbol{x}$ $(\boldsymbol{input})$ is 0?

Solution 3: From $(0, -2)$ the output is -2 or $y = -2$

Vertical Line Test for Functions

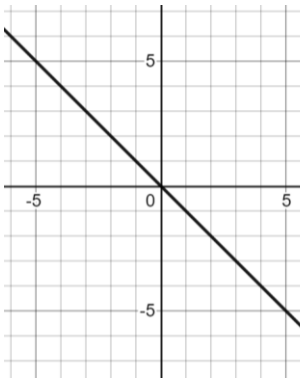
- An equation defines y as a function of x **if and only if** every **vertical line** in the coordinate plane **intersects the graph** of the equation **only once**.

Horizontal Line Test for One-to-One Functions

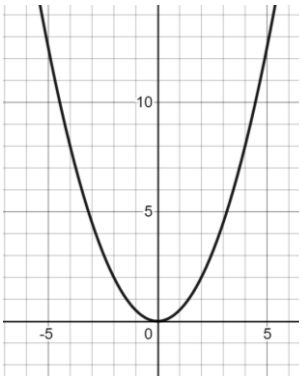
- A function y is a one-to-one function of x **if and only if** every **horizontal line** in the coordinate plane **intersects the function** at most **only once**.

Example 4: State whether the following relations is a function, a one-to-one function, or neither.

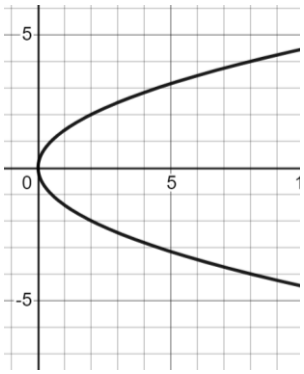
a)



b)



c)



Solution 4:

a) A **vertical line** intersects the graph **once** so it is a **function**. A **horizontal line** intersects the graph **once**, therefore it is a **one-to-one function**.

b) A **vertical line** intersects the graph **once**, so it is a **function**. A **horizontal line** intersects the graph **more than once**, therefore the graph is **not a one-to-one function**.

c) A **vertical line** intersects the graph **more than once**, so it is **not a function**, just a **relation**.

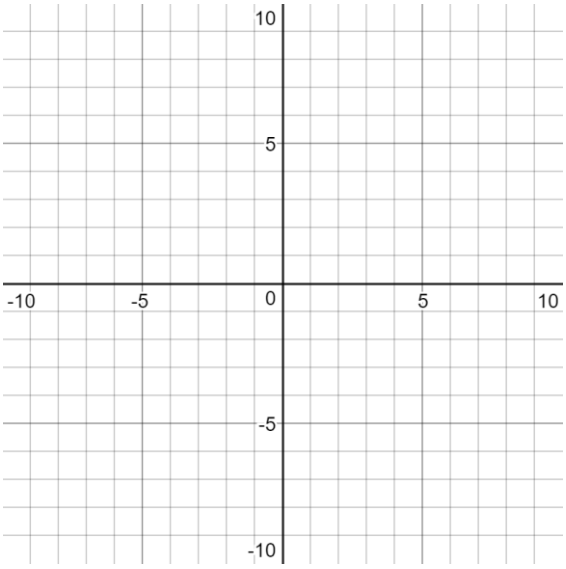
Section 2.1a – Practice Questions

Without plotting on a grid, which quadrant do the following points belong to?

1. $(4, -2)$	2. $(6, 3)$
3. $(-1, 3)$	4. $(-2, -6)$
5. $(-3, 0)$	6. $(0, 0)$

7. Plot the points of the grid provided

$A(-3, 1)$	$B(-4, -2)$	$C(-5, 0)$	$D(0, 2)$
$E(3, -5)$	$F(4, 3)$	$G(4, 0)$	$H(0, -4)$

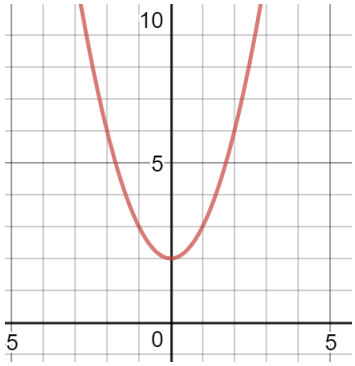


8. A relation is:
   a) Any set of ordered pairs
   b) Two sets of ordered pairs that are related
   c) A graph of ordered pairs
   d) A set of ordered pairs where the domain corresponds to exactly one range

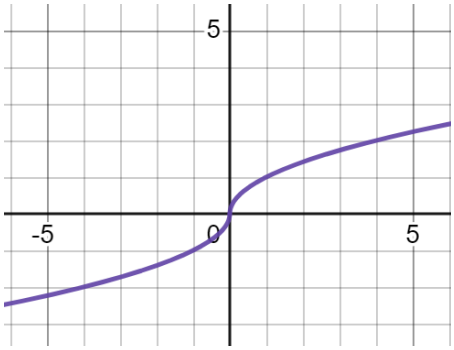
9. A function is:
   a) Any set of ordered pairs
   b) A set or ordered pairs in which a value in the domain corresponds to exactly one value in the range
   c) A set of ordered pairs in which a value in the range corresponds to exactly one value in the domain
   d) A graph of ordered pairs

Use the vertical line test to determine if the following are relations or functions

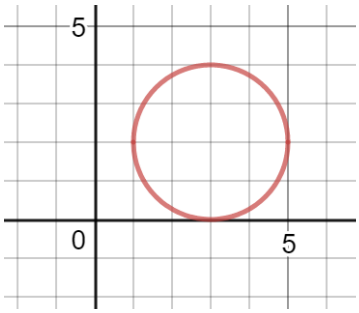
10.



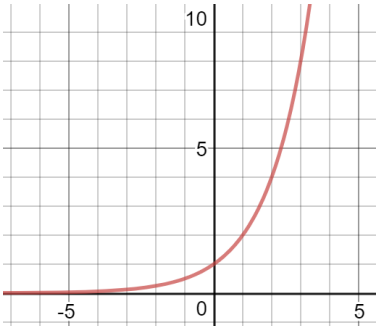
11.



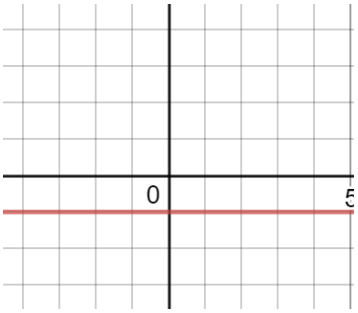
12.



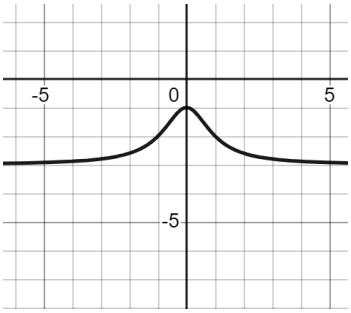
13.



14.

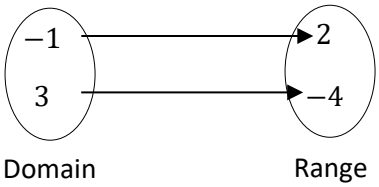


15.

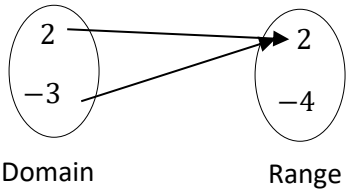


Do the mapping notations into functions, 1-1 functions, or neither?

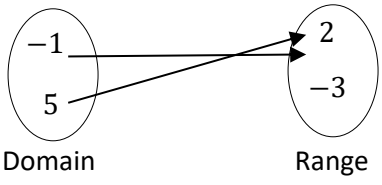
16.



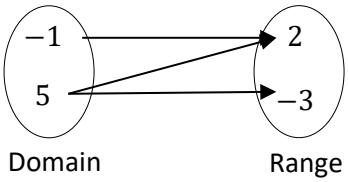
17.



18.



19.



Section 2.1a – Answer Key

1. IV
2. I
3. II
4. III
5. No Quadrant
6. No Quadrant
7. See Website
8. a
9. b
10. Function
11. Function
12. Not a Function
13. Function
14. Function
15. Function
16. $1 - 1$
17. Function
18. Function
19. Neither

Extra Work Space