

## Section 2.1b – Domain, Range, and Non-Linear Equations

This booklet belongs to: \_\_\_\_\_ Block: \_\_\_\_\_

### Expressions

- An expression is a collection of numbers, variables and operation signs
- Expressions **DO NOT** have an **EQUALS SIGN**

### Examples:

- a) 5
- b)  $2x - 3$
- c)  $3x^2 + 2x - 5$
- d)  $\sqrt{5}$

### Equations

- An equation is a mathematical statement that two expressions are equivalent
- There is an **EQUALS SIGN**

### Examples:

- a)  $y = 2$
- b)  $y = 3x + 4$
- c)  $x + 4y = 7$
- d)  $x = y^2$

### Domain and Range

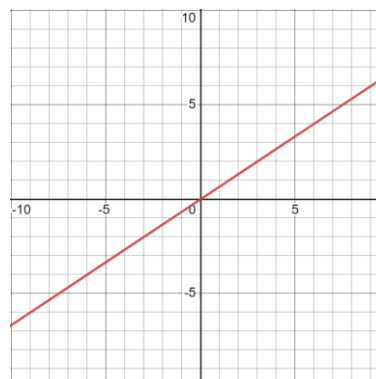
- Set Notation is how we express Domain and Range
- It tells us what values are included in our Domain and Range
- Consider the following graph,
- It extends infinitely positive and negative on both the x-axis and y-axis
- So, we say:

$$D: \{x \mid x \in \mathbb{R}\}$$

x, such that, x is in the set of all real numbers

$$R: \{y \mid y \in \mathbb{R}\}$$

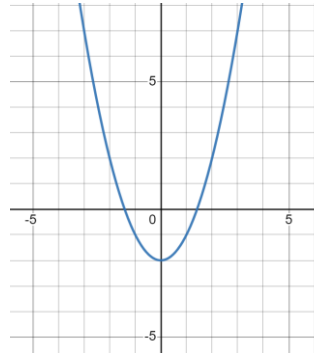
y, such that, y is in the set of all real numbers



- Consider the following graph
- In this one we see that graph will extend infinitely along the x-axis
- The y-values have a lowest most point, so we have to use inequalities to describe the range

$$D: \{x \mid x \in \mathbb{R}\}$$

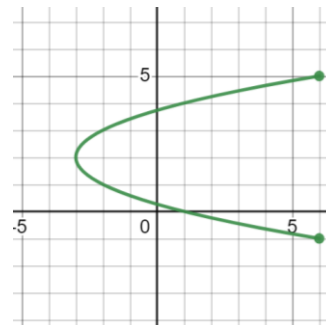
$$R: \{y \mid y \geq -2; y \in \mathbb{R}\}$$



- The following graph has a Domain and Range restriction
- It is bounded by the left and right most point and the upper and lower most point
- It is what we call a 'continuous' function because there are no breaks

$$D: \{x \mid -3 \leq x \leq 6; x \in \mathbb{R}\}$$

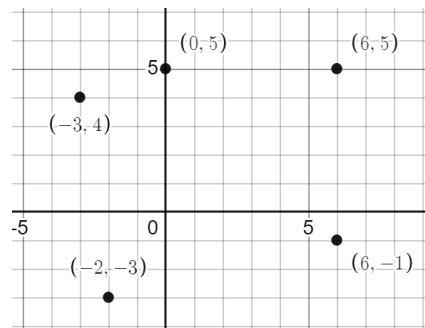
$$R: \{y \mid -1 \leq y \leq 5; y \in \mathbb{R}\}$$



- The following is a graph of discrete points.
- In this case we list only the x-values and y-values of the points
- Do not repeat values if two or more points have the same coordinate

$$D: \{-3, -2, 0, 6\}$$

$$R: \{-3, -1, 4, 5\}$$



- In Grade 11 and 12, we will expand our notation to Interval Notation as well as Set Notation

**Non-Linear Equations – Parabolas Only**

**Rules for graphing Non-Linear Equations**

1. Use **positive numbers, negative numbers, and zero** whenever possible.
2. If **any value is to an even power both positive and negative** values must be used.
3. Try to find the point of symmetry (Where the Parabola reaches it's max/min point)

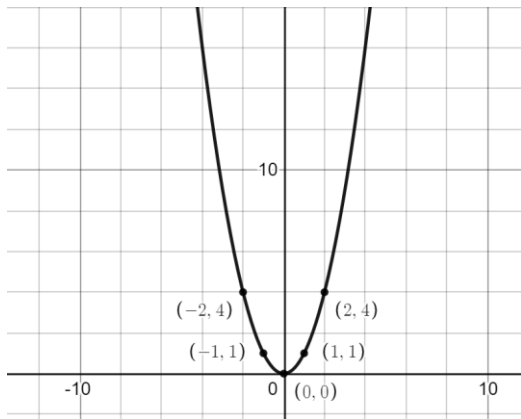
**Example 1:** Graph:  $y = x^2$  What is the Domain and the Range of the Graph?

**Solution 1:** Since  $x$  is an even power, **positive and negative values** of  $x$  are used.

$x$	0	1	-1	2	-2
$y$					

→

$x$	0	1	-1	2	-2
$y$	0	1	1	4	4



Notice the Line of Symmetry occurs at the lowest point, the Vertex of the Parabola!

$x = 0$

Vertical Line through 0 on the  $x - axis$

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D:  $\{x | x \in \mathbb{R}\}$

R:  $\{y | y \geq 0; y \in \mathbb{R}\}$

**Example 2:** Graph:  $y = x^2 - 1$  What is the Domain and the Range of the Graph?

**Solution 2:** Since  $x$  is an even power, **positive and negative values** of  $x$  are used.

$x$	0	1	-1	2	-2
$y$					

→

$x$	0	1	-1	2	-2
$y$	-1	0	0	3	3

Notice the Line of Symmetry occurs at the lowest point, the Vertex of the Parabola!

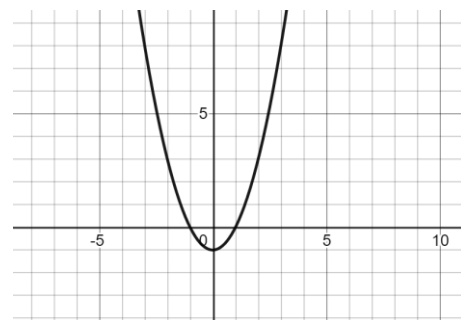
$x = 0$

Vertical Line through 0 on the  $x - axis$

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D:  $\{x | x \in \mathbb{R}\}$

R:  $\{y | y \geq -1; y \in \mathbb{R}\}$



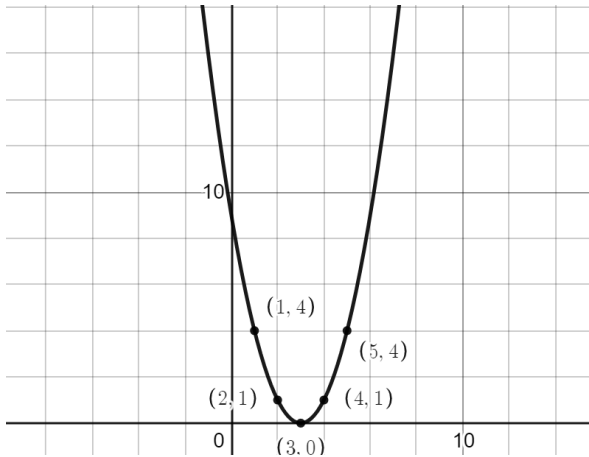
**Example 3:** Graph:  $y = (x - 3)^2$  What is the Domain and the Range of the Graph?

**Solution 3:** Consider what makes the inner portion of the brackets equal zero. Then choose values to the left and right of that.

$x$	3	2	1	4	5
$y$					

→

$x$	3	2	1	4	5
$y$	0	1	4	1	4



Notice the Line of Symmetry occurs at the lowest point, the Vertex of the Parabola!

$$x = 3$$

Vertical Line through 3 on the  $x - axis$

$$D: \{x \mid x \in \mathbb{R}\}$$

$$R: \{y \mid y \geq 0; y \in \mathbb{R}\}$$

**Example 4:** Graph:  $y = -x^2 + 4$  What is the Domain and the Range of the Graph?

**Solution 4:** Remember the negative is not included in the squaring.  $-x^2 \rightarrow (-1)x^2$

$x$	0	1	-1	2	-2
$y$					

→

$x$	0	1	-1	2	-2
$y$	0	3	3	0	0

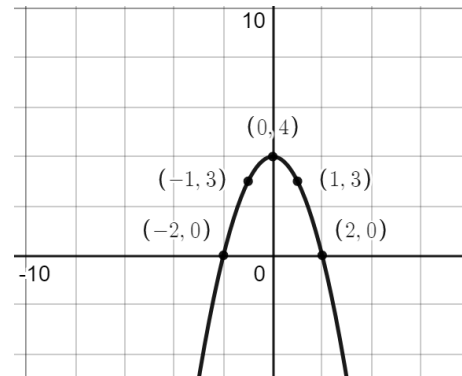
Notice the Line of Symmetry occurs at the highest point, the Vertex of the Parabola!

$$x = 0$$

Vertical Line through 0 on the  $x - axis$

$$D: \{x \mid x \in \mathbb{R}\}$$

$$R: \{y \mid y \leq 4; y \in \mathbb{R}\}$$



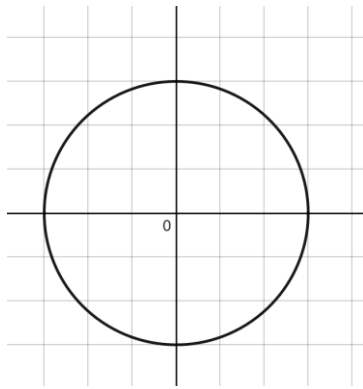
**Just for Fun – A Couple Complicated Examples**

**Example 5:** Is  $x^2 + y^2 = 9$  a function? What is the Domain and Range of the graph?

**Solution 5:** Since  $x$  and  $y$  is an even power, **positive and negative** values of  $x$  and  $y$  are used.

$x$	0		1	-1			3	-3			5	-5	→
$y$		0			1	-1			3	-3			

$x$	0	$\pm 3$	1	-1	$\pm\sqrt{8}$	$\pm\sqrt{8}$	3	-3	0	0	5	-5
$y$	$\pm 3$	0	$\pm\sqrt{8}$	$\pm\sqrt{8}$	1	-1	0	0	3	-3	$\emptyset$	$\emptyset$



The graph is not a function since it does not pass the vertical line test.

D:  $\{x \mid -3 \leq x \leq 3; x \in \mathbb{R}\}$   
 R:  $\{y \mid -3 \leq y \leq 3; y \in \mathbb{R}\}$

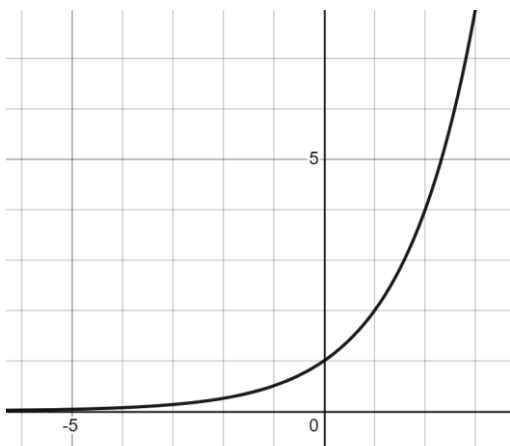
**Example 8:** Is  $y = 2^x$  a function? What is the Domain and the Range of the Graph?

**Solution 8:** Since  $x$  is in the exponent, **positive and negative** values of  $x$  are used

$x$	-3	-2	-1	0	1	2	3
$y$							

→

$x$	-3	-2	-1	0	1	2	3
$y$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



The graph is a function since it does pass the vertical line test.

D:  $\{x \mid x \in \mathbb{R}\}$   
 R:  $\{y \mid y > 0; y \in \mathbb{R}\}$

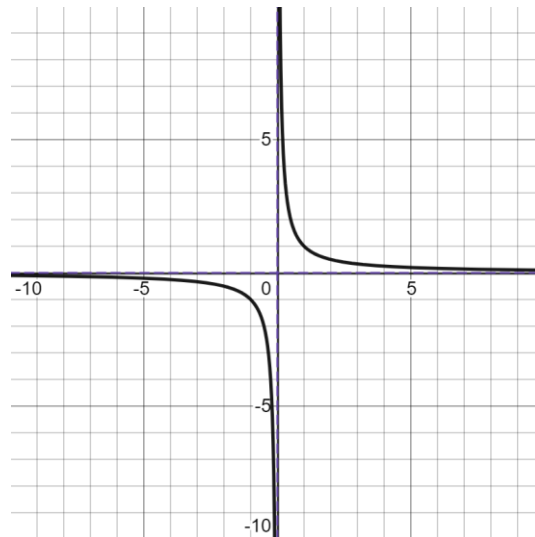
**Example 9:** Is  $y = \frac{1}{x}$  a function? What is the Domain and the Range of the Graph?

**Solution 9:** Since  $x$  is in the denominator, values between 0 and 1 must be used

$x$	-3	-2	-1	0	1	2	3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	→
$y$												

$x$	-3	-2	-1	0	1	2	3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$
$y$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	$\emptyset$	1	$\frac{1}{2}$	$\frac{1}{3}$	2	-2	4	-4

$D: \{x \mid x \neq 0; x \in \mathbb{R}\}$   
 $R: \{y \mid y \neq 0; y \in \mathbb{R}\}$



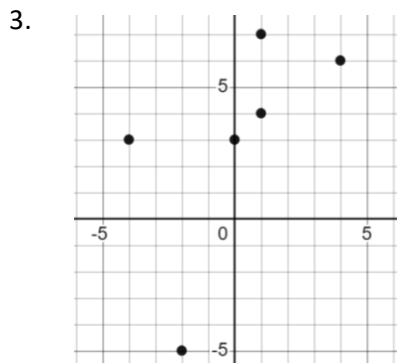
- The **graph is a function** since it does pass the vertical line test.
- $x$  **cannot be zero** because  $\frac{1}{0}$  does not exist (asymptote)
- $y$  **cannot be zero** because 1 divided by a very large number is a very small number, but still not zero.

## Section 2.1b – Practice Questions

### EMERGING LEVEL QUESTIONS

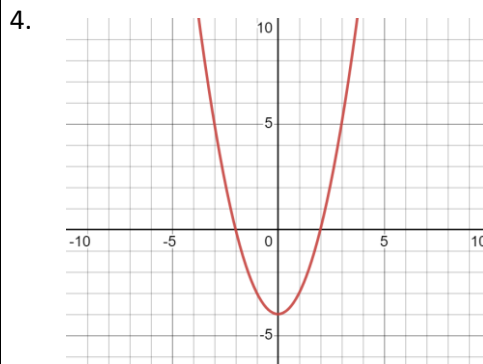
1. The domain of a relation is:
  - a) The set of all  $x$  and  $y$  values in ordered pairs
  - b) The sum of the components in the ordered pairs
  - c) The set of all the first components in the ordered pairs
  - d) The set of all the second components on the ordered pairs
  
2. The range of a relation is:
  - a) The set of all  $x$  and  $y$  values in ordered pairs
  - b) The sum of the components in the ordered pairs
  - c) The set of all the first components in the ordered pairs
  - d) The set of all the second components on the ordered pairs

State the Domain and Range of the following Graphs



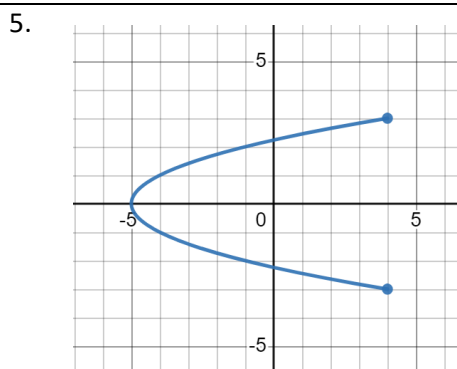
D:

R:



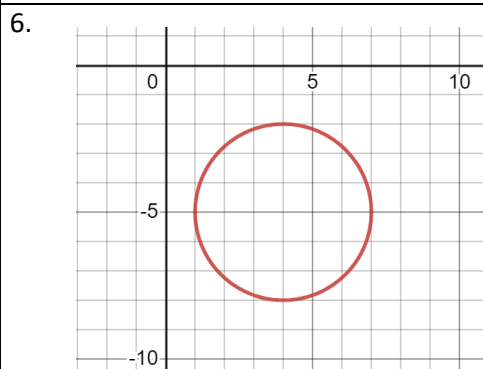
D:

R:



D:

R:



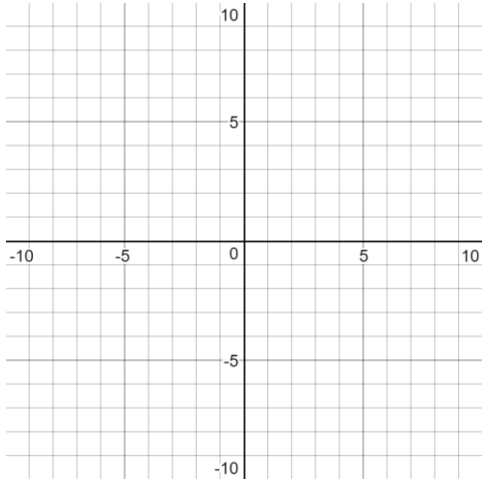
D:

R:

**PROFICIENT LEVEL QUESTIONS**

Graph the Non-Linear Equations, use the table if needed. Include Domain and Range in Set Notation.

7.  $y = x^2 + 1$

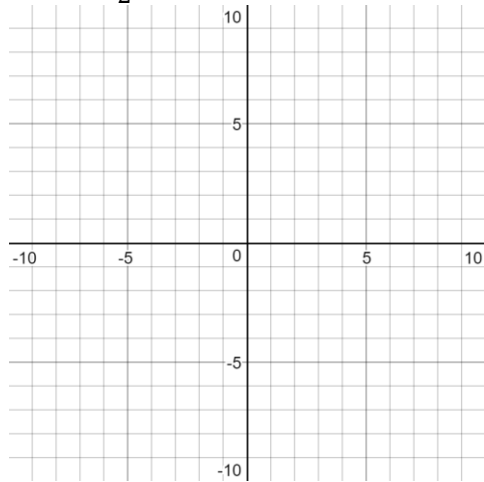


$x$					
$y$					

D:

R:

8.  $y = \frac{1}{2}x^2$

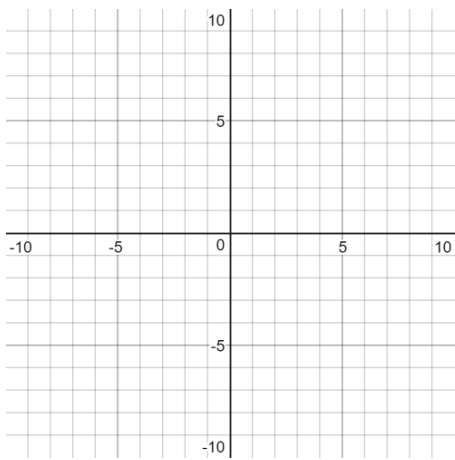


$x$					
$y$					

D:

R:

9.  $y = (x - 2)^2$

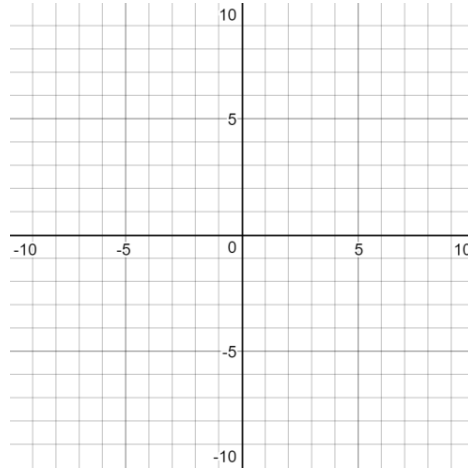


$x$					
$y$					

D:

R:

10.  $y = -x^2$



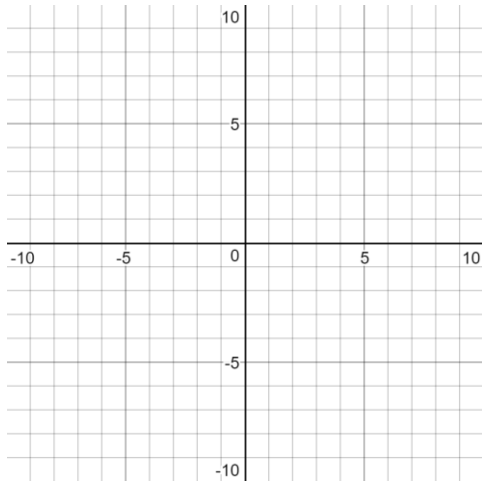
$x$					
$y$					

D:

R:



11.  $y = x^2 - 2$

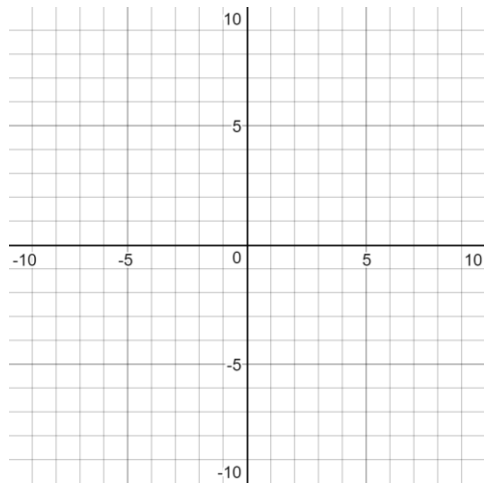


$x$					
$y$					

D:

R:

12.  $y = (x + 2)^2 - 2$



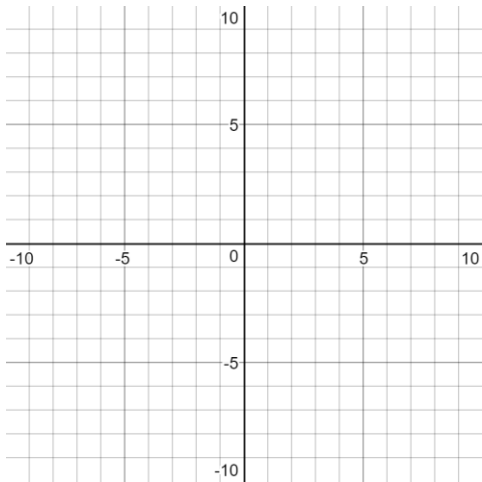
$x$					
$y$					

D:

R:

**EXTENDING LEVEL QUESTIONS**

13.  $y = -x^3$

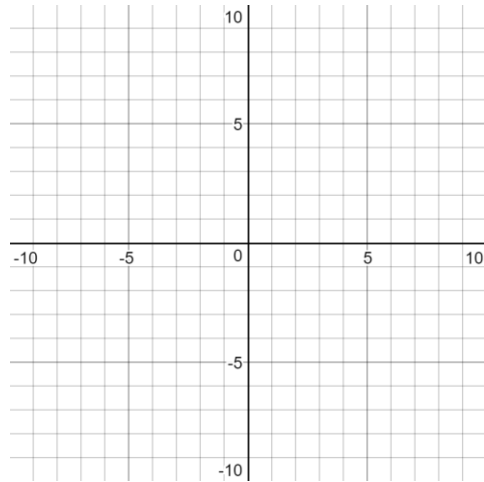


$x$					
$y$					

D:

R:

14.  $x = \frac{1}{2}y^2$



$x$					
$y$					

D:

R:

**See Website for Answer Key – Section 2.1b**

**Extra Work Space**