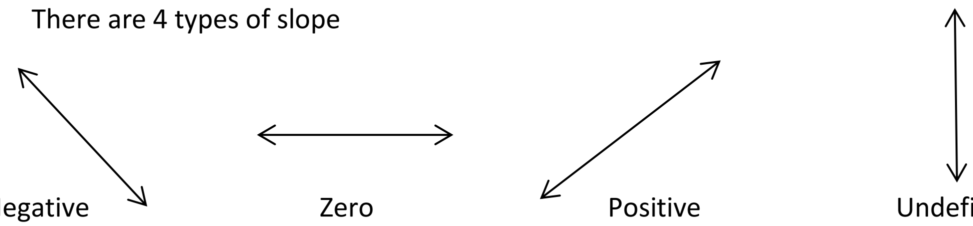
Section 2.2b – Slope and Rate of Change

This booklet belongs to:_____________________________Block: ______

Slope

- The slope of a linear equation describes the steepness and direction of a line
- As a line is traced from left to right the slope is the vertical change relative to the horizontal change
- There are 4 types of slope



Negative Zero Positive Undefined

NOTE: Use Time and Money to Explain Slope

Finding slope from a graph

Slope (m)

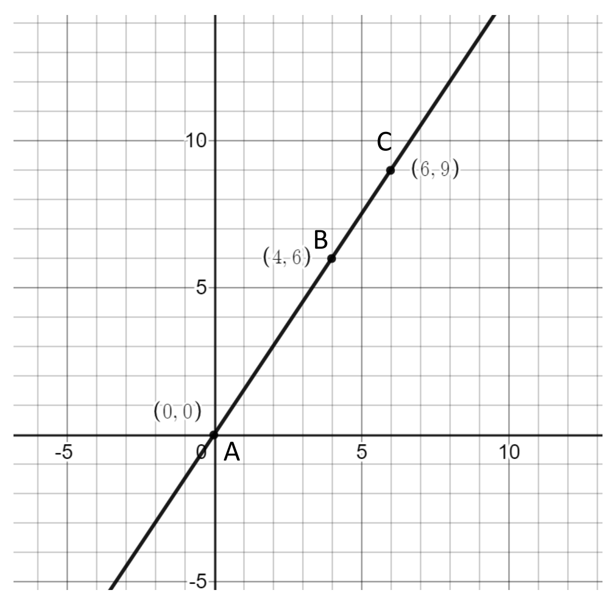
$$m = \frac{vertical\ change}{horizontal\ change} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example of positive slope

Slope of segment AB $= \frac{vertical\ change}{horizontal\ change} = \frac{6}{4} = \frac{3}{2}$

Slope of segment AC $= \frac{vertical\ change}{horizontal\ change} = \frac{9}{6} = \frac{3}{2}$

Slope of segment BC $= \frac{vertical\ change}{horizontal\ change} = \frac{3}{2}$



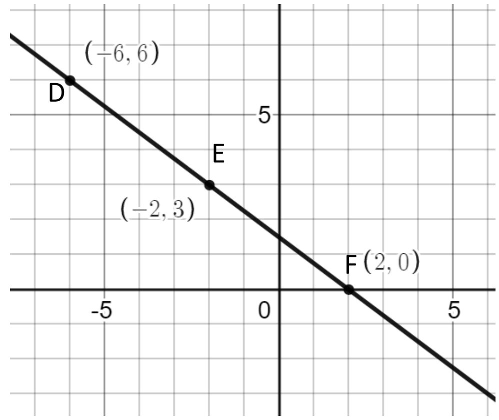
- It doesn't matter where you start measuring, the slope of a straight line is constant between any two points you pick
- The Slope does not change!

Example of negative slope

Slope of segment DE = $\frac{vertical\ change}{horizontal\ change} = -\frac{3}{4}$

Slope of segment DF = $\frac{vertical\ change}{horizontal\ change} = -\frac{6}{8} = -\frac{3}{4}$

Slope of segment EF = $\frac{vertical\ change}{horizontal\ change} = -\frac{3}{4}$



- It doesn't matter where you start measuring, the slope of a straight line is constant between any two points you pick
- The Slope does not change!

Finding Slope from Ordered Pairs

Slope Formula

The slope, m, of a line segment between two points (x_1, y_1) and (x_2, y_2) is

$$\boldsymbol{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

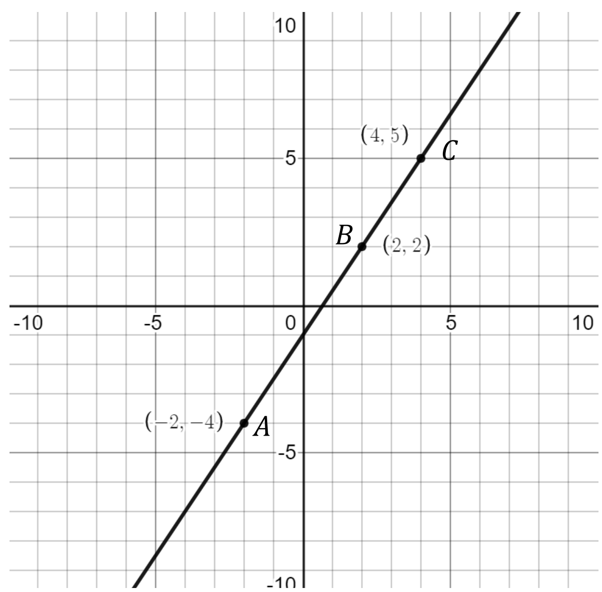
Example of positive slope

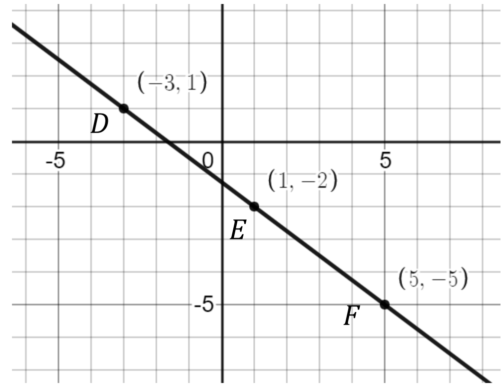
Slope of segment AB = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{-2-2} = \frac{-6}{-4} = \frac{3}{2}$

Slope of segment AC = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-5}{-2-4} = \frac{-9}{-6} = \frac{3}{2}$

Slope of segment BC = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{2-4} = \frac{-3}{-2} = \frac{3}{2}$

- The order you pick the points will not change the outcome. The Slope will be the same either way
- Switch the order and give it a try!





Example of negative slope

Slope of segment DE $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-2)}{-3-1} = \frac{3}{-4} = -\frac{3}{4}$

Slope of segment DF $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-5)}{-3-5} = \frac{6}{-8} = -\frac{3}{4}$

Slope of segment EF $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-5)}{1-5} = \frac{3}{-4} = -\frac{3}{4}$

- The order you pick the points will not change the outcome. The Slope will be the same either way
- Switch the points and give it a try!

Lines with Zero Slope and Undefined Slope

- If two different points have the same $y - value$, the line (or line segment) joining the two points is **horizontal**. (Think h looks like an upside down y)

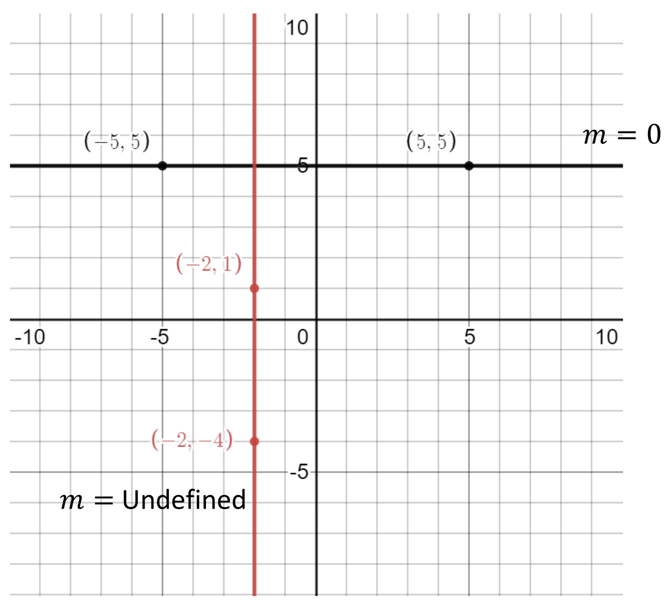
Example of Zero Slope (Black Line on the Grid)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0}{x_2 - x_1} = 0$$

- If two different points have the same $x - value$, the line (or line segment) joining the two points is **vertical**. (Think x looks like two back to back $v's$)

Example of Undefined Slope (Red Line on the Grid)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{0} = \text{Undefined}$$



Rate of Change

- The Greek letter Delta ($\boldsymbol{\Delta}$) is used to represent $\boldsymbol{change}$.
- We use Rates of Changes to help **compare** quantities with different units.
- The formula for Rate of Change is: $\boldsymbol{change\ in\ y}$ **over** $\boldsymbol{change\ in\ x}$.

Rate of Change

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

- Does this look familiar?
- What is the equation for Slope?

Examples of Rates of Change:

1. Kilometers per hour: km/hr or $\frac{km}{h}$
2. Miles per gallon: miles/gal or $\frac{mi}{gal}$
3. Dollars per hour: $/hr or $\frac{dollars}{hour}$
4. If the city of Surrey grew by $120\ 000$ people over a five-year period.

   It has a rate of change of: $\frac{120\ 000\ people}{5\ years} = 24\ 000$ people/year

5. If a person runs the $400m$ race in $56\ seconds$, they run at a rate of:

$$\frac{400m}{56\ sec} = 7.14 \text{ meters/second}$$

- **Rates of Change** are just the slope relationship of two variables
- The variable on the $y - axis$ is the **dependent variable**
- The variable on the $x - axis$ is the **independent variable** (Usually: TIME)

Example 1:

Paul rents a car with a full gas tank. The odometer read 86 347km. Paul used the car for 3 days. When he returned it, the odometer read was 86 721km and it needed 63 litres to fill up. The cost of renting the car was $96 plus gas which cost 90 cents per litre.

a) Determine the fuel efficiency of the car (km/litre).
b) Determine the average rate of travel per day.
c) Determine the cost of renting the car per day.

Solution 1:

a) $\frac{\Delta y}{\Delta x} = \frac{(86\ 721 - 86\ 347)\ km}{(63-0) litres} = $ 5.94 km/litre

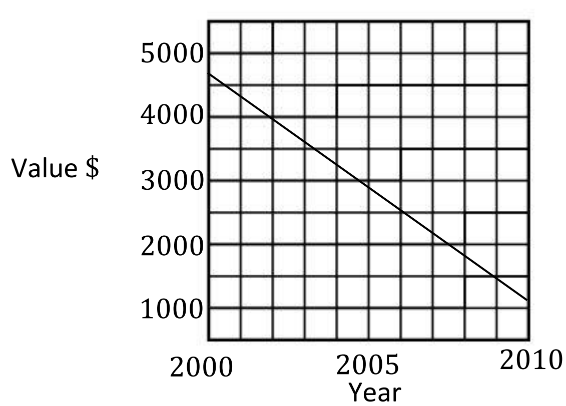
b) $\frac{\Delta y}{\Delta x} = \frac{(86\ 721 - 86\ 347) km}{(3-0) days} = $ 124.7 km/day

c) $\frac{\Delta y}{\Delta x} = \frac{(96-\ 0)\$}{(3-\ 0) days} + \frac{63(0.90)}{3} = \$50.90$/day

- Rates of Change can be visualized using graphs. As mentioned the **denominator** quantity is generally placed of the $\boldsymbol{x - axis}$, the **numerator** value is placed on the $\boldsymbol{y - axis}$.

Example 2: Between 2000 and 2010, the cost of a 42" LCD TV dropped from $4600 to $1200. Graph this result and determine the average drop in price per year.

Solution 2: LCD TVs have been dropping at an average rate of $340 per year.



$$Slope = \frac{\Delta y}{\Delta x} = \frac{4600-1200}{2000-2010} = \frac{\$3400}{-10\ yrs} = \frac{\$340}{-1\ yr} = -\$340/\text{yr}$$

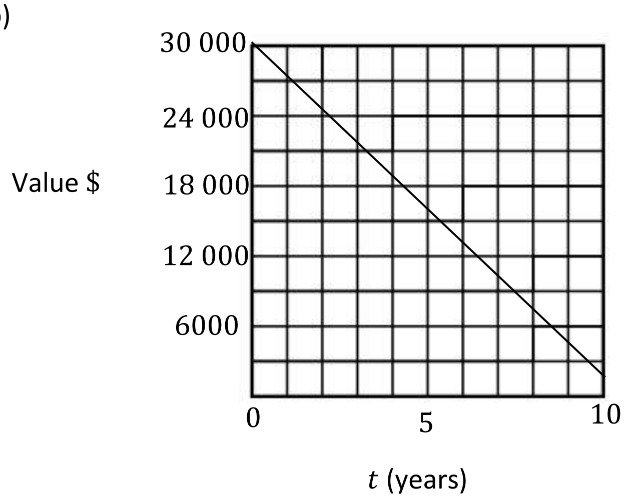
Example 3: Most cars depreciate as they age. A car costing \$30 000 will have a value of \$2500 at the end of 10 years.

a) Write a formula for its value V, when it is t years old. $0 \leq t \leq 10$
b) Draw the graph of this linear function
c) Determine the cars value after 4.5 years
d) When is the car's value between \$12 000 and \$15 000?
e) How much value does the car lose every 2.5 years?
f) What is the rate of change of the car's value with respect to time?

Solution 3:

a) Slope: $\frac{\Delta y}{\Delta x} = \frac{30\,000-2500}{0-10} = \frac{\$27\,500}{-10} = -2750/\text{yr}$ Therefore $V = 30\,000 - 2750t$

b)



c) $V = 30\,000 - 2750(4.5) = \$17\,625$

d) $V = 30\,000 - 2750(t) = 12\,000 \rightarrow 2750t = 18\,000 \rightarrow t = 6.5$ years

$V = 30\,000 - 2750(t) = 15\,000 \rightarrow 2750\,t = 15\,000 \rightarrow t = 5.5$ years

Answers are rounded

e) Since it loses \$2750 per year, $\$2750 \cdot 2.5$ years = \$6875 lost every 2.5 years

f) The rate of change of value is the slope: – \$2750 per year

Example 4:

Georgia sells computers. She is paid a basic monthly salary of $1500, plus $400 for every **five** computers she sells.

a) Write a formula for Georgia's monthly wage.
b) How many computers must be sold for Georgia to make **at least** $3440 in one month?
c) Determine Georgia's wage in a month when she sells 60 computers
d) What is the rate of change of Georgia's wage with respect to the number of computers sold?

Solution 4:

a) $\frac{\Delta y}{\Delta x} = \frac{400-0}{5-0} =$ $80/computer So, $W = 1500 + 80x$

b) $W = 1500 + 80x = 3440 \rightarrow 80x = 1940 \rightarrow x = 24.25$

Georgia must sell 25 computers

c) $W = 1500 + 80(60) = \$6300$

d) The rate of change is the slope: $80 per computer

Example 5:

In the morning, Anna types **nine pages** in 45 minutes. After lunch, she typed **15 pages** in 1 hour and 20 minutes. If the pages typed were approximately the same length, did she type faster in the morning or after lunch?

Solution 5:

Determine the rate in the morning

$\frac{9\ pages}{45\ minutes} =$ 0.2 pages/min

Determine the rate after lunch

$\frac{15\ pages}{80\ minutes} =$ 0.1875 pages/min

Therefore, Anna typed **faster in the morning**.

Section 2.2b – Practice Problems

EMERGING LEVEL QUESTIONS

Fill in the blank with the appropriate word

1. The run between two points on a coordinate system refers to change in the __________ variable
2. The rise between two points on a coordinate system refers to change in the __________ variable
3. The letter ________ is used to indicate the slope of a line
4. The formula for finding the slope of a line is ___________
5. The slope of a vertical line is ___________
6. The slope of a horizontal line is ____________
7. A ___________ has **both** the x-coordinates **and** y-coordinates increasing
8. A ___________ has **either** the x-coordinates **or** y-coordinates decreasing
9. _____________ represents a rate of change

10. Match the column on the left with the column on the right

a) rise	i) $x = 3$
b) run	ii) difference in x
c) slope	iii) $\frac{difference\ in\ y}{difference\ in\ x}$
d) vertical line	iv) difference in y
e) horizontal line	v) $y = -1$

Determine if the slope is positive, negative, zero, or undefined.

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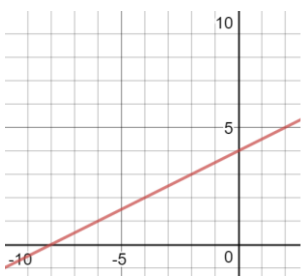

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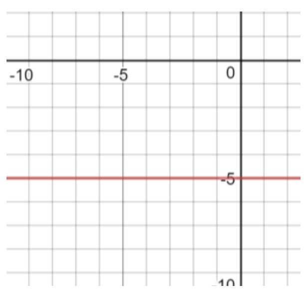

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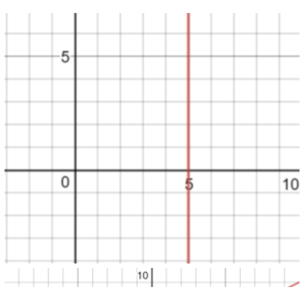

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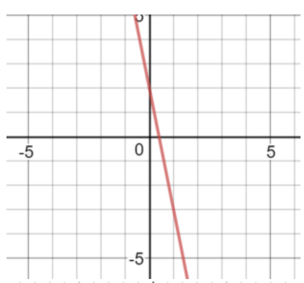
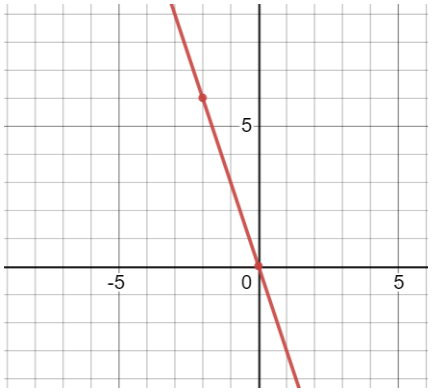


Determine the slope of the line

15.



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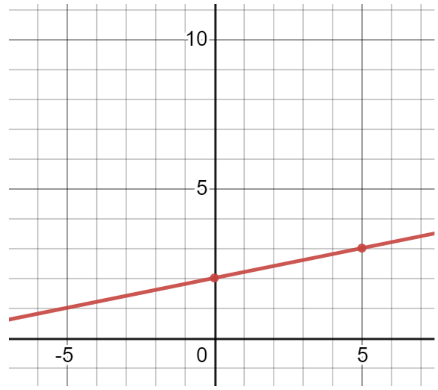

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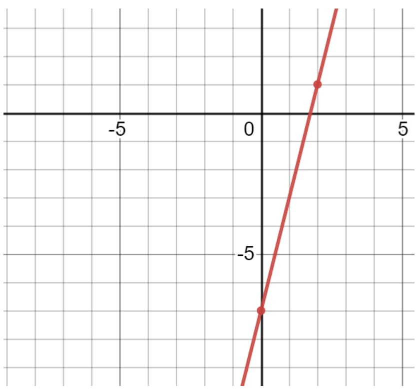

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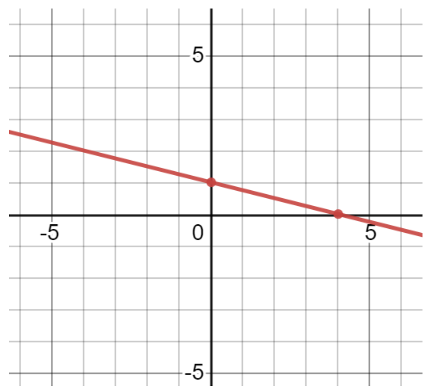

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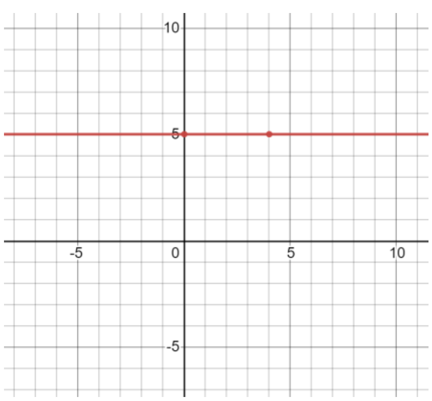

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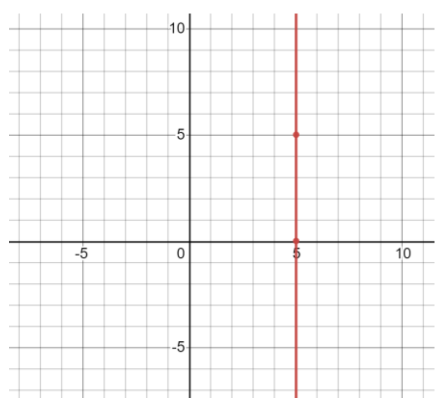

PROFICIENT LEVEL QUESTIONS

Find the slope from the points provided

21. $(2, 3)$ and $(6, 9)$

22. $(3, 2)$ and $(7, 10)$

23. $(-1, 5)$ and $(4, 1)$

24. $(2, 2)$ and $(2, -2)$

25. $(2, -1)$ and $(-5, -1)$

26. $(-3, 1)$ and $(6, 8)$

EXTENDING LEVEL QUESTIONS

27. A long-distance runner passes the 24km mark of a race in 1hr 20 min, and passes the 42km mark 1 hour later. Assuming a constant rate, find the speed of the long-distance runner in km/hr.

28. A plane at an altitude of 20 000 feet starts to descend for landing after flying for six hours. The entire flight time was 6 hours and 40 minutes. Determine the average rate of descent of the plane in ft/min.

29. As a window washer begins work on a high rise with 120 windows, one-third of the windows were already clean. Eight hours later, three-quarters of all the windows are clean. Calculate the window washer's cleaning rate in windows per hour.

30. A five-foot-long treadmill rises 15 inches to make an incline for running. What is the slope of the treadmill?

Section 2.2b – Answer Key

1. x
2. y
3. m
4. $\frac{y_2-y_1}{x_2-x_1}$
5. Undefined
6. 0
7. Positive
8. Negative
9. Slope
10.
    a) iv)
    b) ii)
    c) iii)
    d) i)
    e) v)
11. Positive
12. 0
13. Undefined
14. Negative
15. $m=-3$
16. $m=\frac{1}{5}$
17. $m=4$
18. $m=-\frac{1}{4}$
19. $m=0$
20. Undefined
21. $m=\frac{3}{2}$
22. $m=2$
23. $m=-\frac{4}{5}$
24. Undefined
25. $m=0$
26. $m=\frac{7}{9}$
27. 18km/hr
28. 500ft/min
29. 6.5window/hr
30. 0.26inch/inch