Section 2.3 – Practice Problems

EMERGING LEVEL QUESTIONS

Write the first four terms of each of the following sequences

1. $\{n^2 - 2\}$

$n=1 \quad \{1^2-2\} = \boxed{-1}$

$n=2 \quad \{2^2-2\} = \boxed{2}$

$n=3 \quad \{3^2-2\} = \boxed{7}$

$n=4 \quad \{4^2-2\} = \boxed{14}$

2. $\left\{\frac{n+2}{n+1}\right\}$

$n=1 \quad \left\{\frac{1+2}{1+1}\right\} = \boxed{\frac{3}{2}}$

$n=2 \quad \left\{\frac{2+2}{2+1}\right\} = \boxed{\frac{4}{3}}$

$n=3 \quad \left\{\frac{3+2}{3+1}\right\} = \boxed{\frac{5}{4}}$ $\quad n=4 \quad \left\{\frac{4+2}{4+1}\right\} = \boxed{\frac{6}{5}}$

3. $\{(-1)^{n+1}n^2\}$

$n=1 \quad \{(-1)^2(1)^2\} = \boxed{1}$

$n=2 \quad \{(-1)^3(2)^2\} = \boxed{-4}$

$n=3 \quad \{(-1)^4(3)^2\} = \boxed{9}$

$n=4 \quad \{(-1)^5(4)^2\} = \boxed{-16}$

4. $\left\{\frac{3^n}{2^n+1}\right\}$

$n=1 \quad \left\{\frac{3^1}{2^1+1}\right\} = \frac{3}{3} = \boxed{1}$

$n=2 \quad \left\{\frac{3^2}{2^2+1}\right\} = \boxed{\frac{9}{5}}$ $\quad n=3 \quad \left\{\frac{3^3}{2^3+1}\right\} = \frac{27}{9} = \boxed{3}$

$n=4$

5. $\left\{\frac{2^n}{n^2}\right\}$

$n=1 \quad \left\{\frac{2^1}{1^2}\right\} = \boxed{2}$ $\quad n=2 \quad \left\{\frac{2^2}{2^2}\right\} = \boxed{1}$

$n=3 \quad \left\{\frac{2^3}{3^2}\right\} = \boxed{\frac{8}{9}}$ $\quad n=4 \quad \left\{\frac{2^4}{4^2}\right\} = \boxed{1}$

6. $\left\{\left(\frac{2}{3}\right)^n\right\}$

$n=1 \quad \boxed{\frac{2}{3}}$

$n=2 \quad \boxed{\frac{4}{9}}$ $\quad n=3 \quad \boxed{\frac{8}{27}}$

$n=4 \quad \boxed{\frac{16}{81}}$

Find the indicated arithmetic term

7. $a = 5, d = 3, find\ t_{12}$

$t_n = a + (n-1)d$

$t_{12} = 5 + (12-1)(3)$

$5 + 11(3)$

$\boxed{t_{12} = 38}$

8. $a = \frac{2}{3}, d = -\frac{1}{4}, find\ t_9$

$t_9 = a + (n-1)d$

$t_9 = \frac{2}{3} + (9-1)\left(-\frac{1}{4}\right)$

$t_9 = \frac{2}{3} + -2$

$t_9 = \frac{2}{3} - \frac{6}{3} = \boxed{-\frac{4}{3}}$

9. $a = -\frac{3}{4}, d = \frac{1}{2}, find\ t_{10}$

$t_{10} = a + (n-1)d$

$t_{10} = -\frac{3}{4} + (10-1)(\frac{1}{2})$

$t_{10} = -\frac{3}{4} + \frac{9}{2}$

$t_{10} = -\frac{3}{4} + \frac{18}{4} = \boxed{\frac{15}{4}}$

10. $a = 2.5, d = -1.25, find\ t_{20}$

$t_{20} = a + (n-1)d$

$t_{20} = 2.5 + (20-1)(-1.25)$

$t_{20} = 2.5 + 19(-1.25)$

$t_{20} = 2.5 - 23.75$

$\boxed{t_{20} = -21.25}$

11. $a = -0.75, d = 0.05, find\ t_{40}$

$t_{40} = a + (n-1)d$

$t_{40} = -0.75 + (40-1)(0.05)$

$t_{40} = -0.75 + 1.95$

$\boxed{t_{40} = 1.2}$

12. $a = -\frac{7}{4}, d = -\frac{2}{3}, find\ t_{37}$

$t_{37} = -\frac{7}{4} + (37-1)(-\frac{2}{3})$

$t_{37} = -\frac{7}{4} + 36(-\frac{2}{3})$

$t_{37} = -\frac{7}{4} - 24$

$t_{37} = -\frac{7}{4} - \frac{96}{4} = -\frac{103}{4} = \boxed{-25.75}$

Find the number of terms in each arithmetic sequence

13. $a = 6, d = -3, t_n = -30$

$t_n = a + (n-1)d$

$-30 = 6 + (n-1)(-3)$

$-6 \quad -6$

$\frac{-36}{-3} = \frac{-3(n-1)}{-3}$

$12 = n - 1$

$+1 \quad +1$

$\boxed{n = 13}$

14. $a = -3, d = 5, t_n = 82$

$t_n = a + (n-1)d$

$82 = -3 + (n-1)(5)$

$+3 \quad +3$

$\frac{85}{5} = \frac{5(n-1)}{5}$

$17 = n - 1$

$+1 \quad +1$

$\boxed{n = 18}$

15. $a = 0.6, d = 0.2, t_n = 9.2$

$t_n = a + (n-1)d$

$9.2 = 0.6 + (n-1)(0.2)$

$-0.6 \quad -0.6$

$\frac{8.6}{0.2} = \frac{0.2(n-1)}{0.2}$

$43 = n - 1$

$+1 \quad +1$

$n = 44$

16. $a = -0.3, d = -2.3, t_n = -39.4$

$t_n = a + (n-1)d$

$-39.4 = -0.3 + (n-1)(-2.3)$

$+0.3 \quad +0.3$

$\frac{-39.1}{-2.3} = \frac{(n-1)(-2.3)}{-2.3}$

$17 = n - 1$

$+1 \quad +1$

$n = 18$

17. $-1, 4, 9, \ldots, 159$

$d = 5$ ↗ ↖ this is t_n

$t_n = a + (n-1)d$

$159 = -1 + (n-1)5$

$+1 \quad +1$

$\frac{160}{5} = \frac{5(n-1)}{5}$

$32 = n - 1$

$+1 \quad +1$

$n = 33$

18. $23, 20, 17, \ldots, -100$

$t_n = a + (n-1)d$

$-100 = 23 + (n-1)(-3)$

$-23 \quad -23$

$\frac{-123}{-3} = \frac{-3(n-1)}{-3}$

$41 = n - 1$

$+1 \quad +1$

$n = 42$

PROFICIENT LEVEL QUESTIONS

Find the first term in the arithmetic sequence

19. 6th term is 10; 18th term is 46

Diff in terms: $18 - 6 = 12$

Diff in number: $46 - 10 = 36$

common diff over 12 terms: $\frac{36}{12} = 3$

$d = 3$

use either

$t_6 = 10$

$n = 6 \quad t_6$

$t_n = a + (n-1)d$

$t_6 = a + (6-1)(3)$

$10 = a + (6-1)(3)$

$10 = a + 15$

$a = -5$

20. 4th term is 2; 18th term is 30

$t_4 = 2$ ↙ ↘ $t_{18} = 30$

$d = \frac{30 - 2}{18 - 4} = \frac{28}{14} = 2$

use either one

$t_{18} = a + (18-1)(2)$

$30 = a + 17(2)$

$30 = a + 34$

$a = -4$

21. 9th term is 23; 17th term is −1

$d = \frac{-1-23}{17-9} = \frac{-24}{8} = -3$

$t_9 = a + (9-1)(-3)$

$23 = a + (8)(-3)$

$23 = a - 24$

$+24 \quad +24$

$a = 47$

22. 5th term is 3; 25th term is −57

$d = \frac{-57-3}{25-5} = \frac{-60}{20} = -3$

$t_{25} = a + (25-1)(-3)$

$-57 = a + 24(-3)$

$-57 = a - 72$

$+72 \quad +72$

$a = 15$

23. 13th term is −3; 20th term is −17

$d = \frac{-17-(-3)}{20-13} = \frac{-14}{7} = -2$

$t_{13} = a + (13-1)(-2)$

$-3 = a + 12(-2)$

$-3 = a - 24$

$21 = a$

24. 11th term is 37; 26th term is 32

$d = \frac{32-37}{26-11} = \frac{-5}{15} = -\frac{1}{3}$

$t_{26} = a + (26-1)\left(-\frac{1}{3}\right)$

$32 = a + 25\left(-\frac{1}{3}\right)$

$32 = a - \frac{25}{3}$

$+25/3 \quad +25/3$

$32 + 8\frac{1}{3} = a$

$a = 40\frac{1}{3}$

$S_n = \frac{n}{2}(a+\ell)$

Find the sum of the arithmetic series

25. $3 + 5 + 7 + \cdots + (2n + 1)$

$S_n = \frac{n}{2}(3 + (2n+1))$

$\frac{n(2n+4)}{2} = \frac{2n^2+4n}{2} = \boxed{n^2+2n}$

26. $-1 + 2 + 5 + \cdots + (3n - 4)$

$S_n = \frac{n}{2}(-1 + 3n - 4)$

$S_n = \frac{n}{2}(3n-5)$

$\boxed{S_n = \frac{3n^2-5n}{2}}$ or $\boxed{S_n = \frac{3n^2}{2} - \frac{5n}{2}}$

27. $2 + 5 + 8 + \cdots + 77$

$S_n = \frac{n}{2}(2+77)$

$S_{26} = 13(79)$

$\boxed{S_{26} = 1027}$

need n ↑

use $t_n = a+(n-1)d$

$77 = 2+(n-1)(3)$

$75 = 3(n-1)$

$25 = n-1$

$n = 26$

28. $5 + 9 + 13 + \cdots + 97$

$S_{24} = 12(5+97)$

$= 12(102)$

$\boxed{S_{24} = 1224}$

$t_n = a+(n-1)d$

$97 = 5+(n-1)(4)$

$92 = 4(n-1)$

$23 = n-1$

$n = 24$

29. $(-41) + (-35) + (-29) + \cdots + 541$

$S_{98} = \frac{98}{2}(-41+541)$

$= 49(500)$

$\boxed{S_{98} = 24\,500}$

$t_n = a+(n-1)d$

$541 = (-41)+(n-1)(6)$

$582 = 6(n-1)$

$97 = n-1$

$n = 98$

30. $2\sqrt{5} + 6\sqrt{5} + 10\sqrt{5} + \cdots + 50\sqrt{5}$

$S_{13} = \frac{13}{2}(2\sqrt{5} + 50\sqrt{5})$

$S_{13} = \frac{13}{2}(52\sqrt{5})$

$\boxed{S_{13} = 338\sqrt{5}}$

$t_n = a+(n-1)(4\sqrt{5})$

$50\sqrt{5} = 2\sqrt{5} + 4\sqrt{5}(n-1)$

$48\sqrt{5} = 4\sqrt{5}(n-1)$

$12 = n-1$

$n = 13$

31. $39 + 33 + 27 + \cdots + (-15)$

$S_{10} = \frac{10}{2}(39+(-15))$

$5(24)$

$\boxed{S_{10} = 120}$

$t_n = a+(n-1)(-6)$

$-15 = 39-6(n-1)$

$-54 = -6(n-1)$

$9 = n-1$

$n = 10$

32. $23 + 19 + 15 + ,,, + (-305)$

$S_{83} = \frac{83}{2}(23+(-305))$

$= \frac{83}{2}(-282)$

$\boxed{S_{83} = -11703}$

$t_n = a+(n-1)d$

$-305 = 23+(n-1)(-4)$

$-328 = -4(n-1)$

$82 = n-1$

$n = 83$

EXTENDING LEVEL QUESTIONS

Find the indicated value using the information given

33. $S_{20}, if\ a_1 = 8, a_{20} = 65$

↑ this is l

$S_{20} = \frac{20}{2}(8+65)$

$= 10(73)$

$S_{20} = 730$

34. $S_{21}, if\ a_1 = 8, a_{20} = 65$

need d

$\frac{\text{number}}{\text{term}} = \frac{65-8}{20-1} = \frac{57}{19}$

$d = 3$

therefore

$l = 65+3$

$= 68$

$S_{21} = \frac{21}{2}(8+68)$

$= \frac{21}{2}(76)$

$S_{21} = 798$

35. $S_{56}, if\ a_{56} = 13, d = -9$

need a

$t_n = a + (n-1)d$

$t_n = 13$

$n = 56$

$d = -9$

$13 = a + (56-1)(-9)$

$13 = a + 55(-9)$

$13 = a - 495$

$+495 \quad +495$

$a = 508$

$S_{56} = \frac{56}{2}(508+13)$

$= 28(521)$

$S_{56} = 14\,588$

36. $n\ if\ S_n = 180, a_1 = 4, t_n = 16$

↑ same as l

$S_n = \frac{n}{2}(a+l)$

$180 = \frac{n}{2}(4+16)$

$180 = 20\left(\frac{n}{2}\right)$

$\frac{180}{10} = \frac{10n}{10}$

$n = 18$

37. $d, if\ S_{40} = 680, a_1 = 11$

use $S_n = \frac{n}{2}(2a + (n-1)d)$

$n = 40$
$a = 11$
$S_n = 680$

$680 = \frac{40}{2}(2(11) + (40-1)d)$

$\frac{680}{20} = \frac{20(22 + 39d)}{20}$

$34 = 22 + 39d$

$-22 \quad -22$

$12 = 39d$

$\frac{12}{39} = \frac{39d}{39}$

$d = \frac{12}{39}$

$d = \frac{4}{13}$

38. $S_{62}, if\ a_1 = 10, d = 3$

$S_{62} = \frac{62}{2}(2(10) + (62-1)(3))$

$= 31(20 + 61(3))$

$= 31(203)$

$S_{62} = 6293$

39. $S_{19}, if\ d = 4, a_{19} = 17$

$S_{19} = \frac{19}{2}(-55 + 17)$

$S_{19} = \frac{19}{2}(-38)$

$S_{19} = -361$

↑ from here we can get a.

$t_{19} = a + (n-1)d$

$17 = a + (19-1)(4)$

$17 = a + 18(4)$

$17 = a + 72$

$17 - 72 = a$

$a = -55$

40. $S_{40}, if\ d = -3, a_{40} = 65$

$S_{40} = \frac{40}{2}(182 + 65)$

$S_{40} = 20(247)$

$S_{40} = 4940$

↑ can get a_1

$t_{40} = a + (n-1)d$

$65 = a + (40-1)(-3)$

$65 = a + 39(-3)$

$65 = a - 117$

$117 \quad +117$

$182 = a$