Section 3.1a – The Three Forms of Linear Equations

This booklet belongs to:_____**Block:**_____

Standard Form and General Form of a Linear Equation

- If $A, B, and\ C$ are real numbers, the equation $Ax\ +\ By\ =\ C$ is called the **STANDARD FORM** of the equation of a line.
- It is **best to write** the equation with $A, B, and\ C$ **as integers**, and $A\ \geq\ 0$ (Not Negative).
- **Standard Form** was introduced in Grade 9 and won't be used much longer
- The **GENERAL FORM**: $Ax + By + C = 0$ is a more appropriate form since moving forward we will want our **Linear Equations equal to 0**
- So, the **transformation** from **Standard to General** Form is a simple one:

  - Get **ALL THE TERMS** on the **same side** of the **equal sign**

Standard Form	General Form
$Ax + By = C$	$Ax + By + C = 0$
$A, B, C\ are\ Integers,\ \ A > 0$	$A, B, C\ are\ Integers,\ \ A > 0$

Example 1:   Write the following in **General Form**

$$-3x\ +\ y\ =\ 4 \qquad and \qquad \frac{2}{3}x\ +\ 2y\ =\ 3$$

Solution 1:

$-3x\ +\ y\ =\ 4$	can be expressed as:	$3x - y\ =\ -4 \leftarrow$ multiply all terms by (-1)
$3x - y = -4$	can be expressed as:	$3x - y + 4 = 0 \leftarrow$ add 4 to both sides

$\frac{2}{3}x\ +\ 2y\ =\ 3$	can be expressed as:	$2x\ +\ 6y\ =\ 9 \leftarrow$ multiply each term by (3)
$2x\ +\ 6y\ =\ 9$	can be expressed as:	$2x + 6y - 9 = 0 \leftarrow$ subtract 9 from both sides

Slope-Intercept Form of a Linear Equation

- The equation $y = mx + b$ is the **SLOPE-INTERCEPT FORM** of the equation of a line.
- The $y-intercept$ of the line is $(0, b)$, and the $slope$ of the line is m.
- The algebra of **STANDARD FORM** to **SLOPE-INTERCEPT FORM** is as follows:

$$Ax + By = C \qquad \rightarrow \qquad By = -Ax + C \qquad \rightarrow \qquad y = -\frac{A}{B}x + \frac{C}{B}$$

- The $slope$ of $Ax + By = C$ is $-\frac{A}{B}$

- The $y-intercept$ of $Ax + By = C$ is $\frac{C}{B} \rightarrow (0, \frac{C}{B})$

SLOPE-INTERCEPT FORM
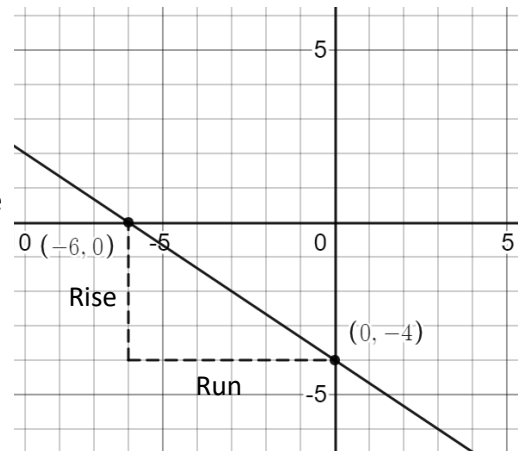
$$y = mx + b$$

Example 2: Convert $-2x - 3y = 12$ to **Slope-Intercept Form** and then Graph the Linear Equation

Solution 2:

$$-2x - 3y = 12 \qquad \text{so...} \quad -3y = 2x + 12 \qquad \qquad \text{so...} \quad y = \frac{2x + 12}{-3}$$

$$\text{so...} \quad y = -\frac{2}{3}x - 4$$

- The $slope$ of the line is: $\frac{2}{3}$ and the $y-intercept$ is $(0, -4)$

- Once you do the algebra it's easy to map
- Plot the y-intercept
- Trace your slope
- Map the next point
- Continue that process and connect the points with a line

Graphing a Line Using the $Slope$ and the $y - Intercept$

Step 1: Write the equation in **SLOPE INTERCEPT FORM** by solving for y
Step 2: Identify the $y - intercept$ $(0, b)$, and graph this point
Step 3: Graph a **second point using the $slope$**, starting at the $y - intercept$
Step 4: Draw a line connecting the points to obtain the graph

Example 3:

Graph: $3x + 2y = 12$ by using the $slope$ and $y - intercept$

Solution 3:

Step 1: $3x + 2y = 12$ so… $2y = -3x + 12$ so… $y = -\frac{3}{2}x + 6$
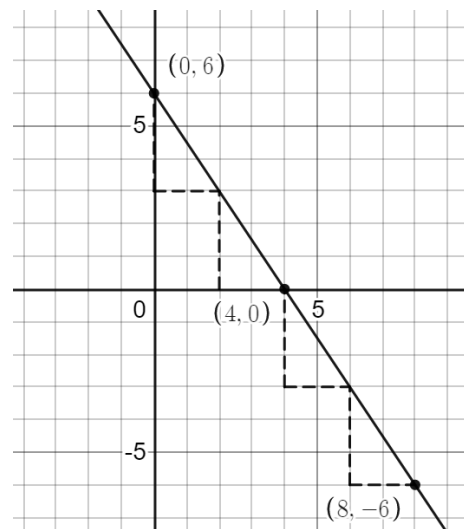
Step 2:   The $y - intercept$ is $(0, 6)$: mark this point

Step 3:   The slope is $m = \frac{rise}{run} = -\frac{3}{2}$

From $(0, 6)$, go $down$ 3 units   and to the $right$ 2 units, to obtain the point $(2, 3)$.

Repeat that step as many times as you want.

Step 4:   Draw a line through the points
                    $(0, 6), (2, 3)$ and $(4, 0)$.

A Note About Slope

When you have a slope, it's a fraction.

So, if it's **negative**: $-\frac{3}{2} = \frac{-3}{2} = \frac{3}{-2}$

- You can go **DOWN and RIGHT** or **UP and LEFT**

So, if it's **positive**: $\frac{3}{2} = \frac{3}{2} = \frac{-3}{-2}$

- You can go **UP and RIGHT** or **DOWN and LEFT**

**Graphing a Line Using the Slope and a Point**

Step 1:   Locate and graph the **given point**.
Step 2:   Graph another point **tracing the slope**, counting from the first point
Step 3:   Repeat that step again
Step 4:   Draw a line connecting the three points to obtain the graph

Example 4:

Graph the line through $(-2, -4)$ with $slope\ \frac{3}{1}$.

Solution 4:

The $slope$ is 3, so from the point $(-2, -4)$ go **$up\ 3\ units$**, and to the **$right\ 1\ unit$** to get the next point $(-1, -1)$. **Repeat tracing the slope** from the new point $(-1, -1)$.
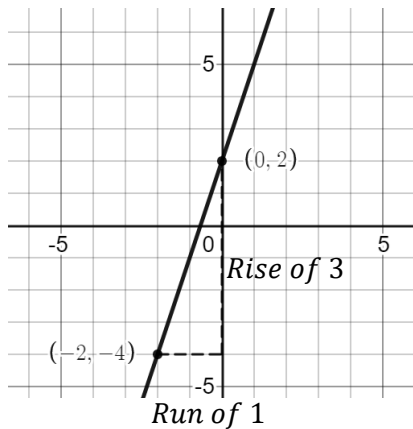
Writing an Equation of a Line Using a Slope and a Point

- By **substituting** given **values** for a $slope\ and\ a\ point\ (x, y)\ of\ a\ line$ into $y = mx + b$, the line's equation can be found!

Example 5:

Write the equation of the line with $slope$ 2 that runs through $(-4, 1)$ $in\ slope\ intercept - form.$

Solution 5:

The point $(-4, 1)$ gives us an $x - value$ of -4 and a $y - value$ of 1.

So, $y = mx + b \rightarrow$ $1 = 2(-4) + b$

                                                          $1 = -8 + b$

                                                          $b = 9$

Therefore, the equation of the line is… $\boldsymbol{y = 2x + 9}$

Point-Slope Form of a Linear Equation

- The equation $y - y_1 = m(x - x_1)$ is the **POINT-SLOPE EQUATION** of a line.
- The **given point** is (x_1, y_1) and the **slope** of the line is m
- This formula comes from rearranging the definition of the slope, $m = \frac{y - y_1}{x - x_1}$

POINT-SLOPE EQUATION

$$y - y_1 = m(x - x_1)$$

Example 6:

Write the equation of a line with $slope$ 2 that passes through $(-4, 1)$ in **Slope-intercept form**.

Solution 6: $y - y_1 = m(x - x_1)$ → $y - 1 = 2(x - (-4))$

$$y - 1 = 2(x + 4)$$

$$y - 1 = 2x + 8$$

$$\boldsymbol{y = 2x + 9}$$

Example 7:

Write the equation of a line with $slope$ $\dfrac{4}{5}$ that passes through $(3, -2)$ in **Standard form.**

Solution 7: $y - y_1 = m(x - x_1)$ → $y - (-2) = \frac{4}{5}(x - 3)$

$$y + 2 = \frac{4}{5}(x - 3)$$

$$5(y + 2) = 4(x - 3)$$

$$5y + 10 = 4x - 12$$

$$\boldsymbol{4x - 5y = 22}$$

Section 3.1a – Practice Problems

EMERGING LEVEL QUESTIONS

Complete each statement

1. The formula for the **Point-Slope form** of a line is _____

2. In the equation $y = mx + b$, $(\mathbf{0}, \mathbf{b})$ is called the _____

3. The equation $\mathbf{y = mx + b}$ is called the _____ form

4. The **Standard form** of the equation of a line is _____

Find the **slope and the y-intercept**

5. $3x - 2y = 6$

Slope:

y-int:

6. $4x + 3y = 12$

Slope:

y-int:

7. $2x - 5y = -7$

Slope:

y-int:

8. $5x + 2y = 0$

Slope:

y-int:

9. $x - 4y = -4$

Slope:

y-int:

10. $6x - y = -3$

Slope:

y-int:

Rewrite the **Standard Form Equation** in **Slope-Intercept Form**

11. $2x + y = 6$	12. $3x - y = 4$
13. $4x + 3y = 12$	14. $2x - 3y = 6$
15. $5x + 4y = 3$	16. $6x - 3y = 4$

Rewrite the **Slope-Intercept Equation** in **Standard Form**

17. $y = -2x + 1$	18. $y = 3x - 1$

19. $y = 3x$

20. $y = -\frac{2}{3}x + 1$

21. $y = \frac{3}{4}x + 5$

22. $y = -\frac{2}{5}x + \frac{1}{2}$

PROFICIENT LEVEL QUESTIONS

Rewrite the **Point-Slope Equation** in **Slope-Intercept Form**

23. $y - 2 = 3(x + 1)$

24. $y + 4 = -2(x - 1)$

25. $y - 1 = \frac{1}{3}(x + 2)$

26. $y + 4 = -\frac{2}{5}(x - 3)$

27. $y - \frac{2}{3} = \frac{1}{4}(x - 8)$

28. $y - \frac{1}{4} = \frac{1}{2}\left(x + \frac{2}{3}\right)$

Rewrite the **Point-Slope Equation** in **Standard Form**

29. $y - 2 = 3(x + 1)$

30. $y + 4 = -2(x - 1)$

31. $y - 1 = \frac{1}{3}(x + 2)$

32. $y + 4 = -\frac{2}{5}(x - 3)$

33. $y - \frac{2}{3} = \frac{1}{4}(x - 8)$

34. $y - \frac{1}{4} = \frac{1}{2}\left(x + \frac{2}{3}\right)$

EMERGING LEVEL QUESTIONS

Write the equation of each line in **slope-intercept form**

35. $(0, 2); m = 2$

36. $(0, -3); m = \frac{1}{2}$

37. $(0,3); m = 0$

38. $(0,-2); m = -\dfrac{2}{3}$

39. $\left(0, -\dfrac{1}{2}\right); m = -\dfrac{3}{4}$
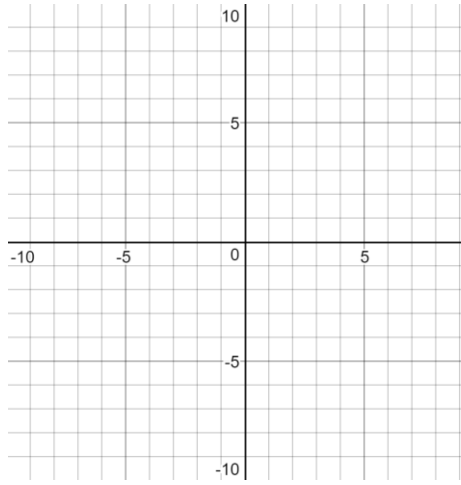
40. $(0, 2.3); m = 0.4$

Graph the Linear Equations

41. $4x - 3y = 12$
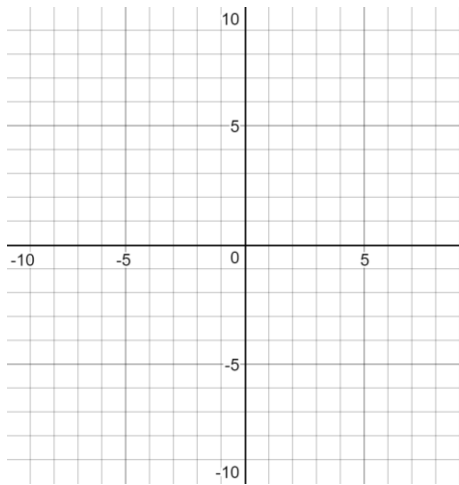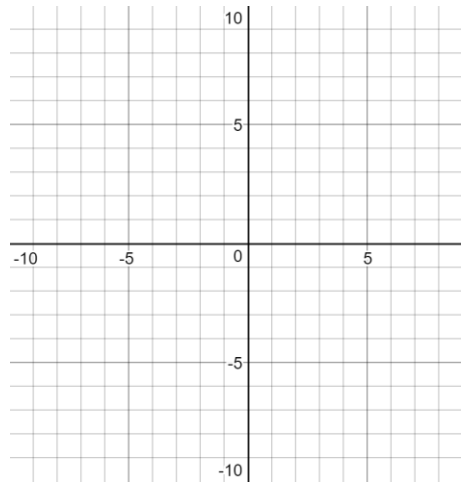

42. $y = -\dfrac{2}{3}x + 4$
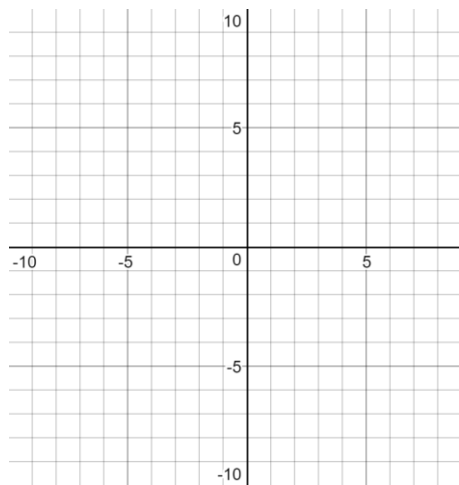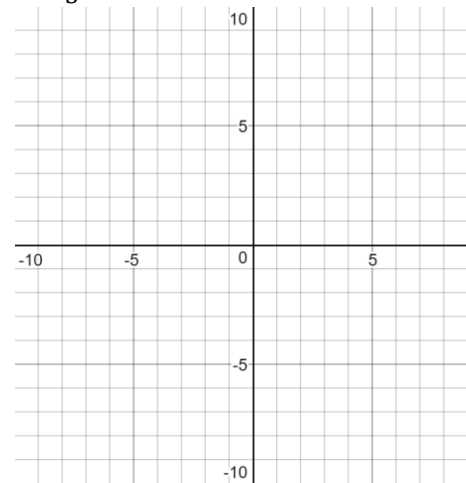
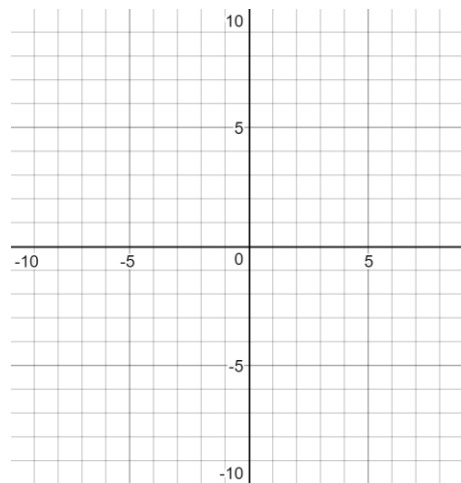**PROFICIENT LEVEL QUESTIONS**

43. $y - 3 = \frac{1}{2}(x + 4)$

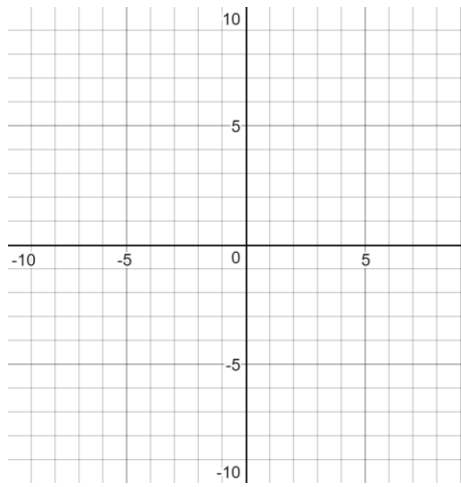44. $2x + 3y = 10$

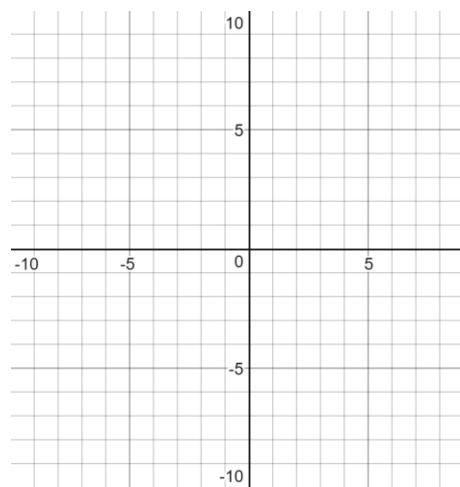45. $y + 2 = -\frac{2}{3}(x + 5)$

46. $5x - 2y = 0$

EXTENDING LEVEL QUESTIONS

47. $y - \dfrac{5}{2} = -\dfrac{1}{2}\left(x + \dfrac{3}{2}\right)$

48. $y = \dfrac{5}{3}x - \dfrac{7}{2}$

Section 3.1a – Answer Key

1. $y - y_1 = m(x - x_1)$	42. See Website
2. y-intercept	43. See Website
3. Slope-Intercept	44. See Website
4. $Ax + By = C$	45. See Website
5. Slope: $\frac{3}{2}$; y-int: -3	46. See Website
6. Slope: $-\frac{4}{3}$; y-int: 4	47. See Website
7. Slope: $\frac{2}{5}$; y-int: $\frac{7}{5}$	48. See Website
8. Slope: $-\frac{5}{2}$; y-int: 0	
9. Slope: $\frac{1}{4}$; y-int: 1	
10. Slope: 6; y-int: 3	
11. $y = -2x + 6$	
12. $y = 3x - 4$	
13. $y = -\frac{4}{3}x + 4$	
14. $y = \frac{2}{3}x - 2$	
15. $y = -\frac{5}{4} + \frac{3}{4}$	
16. $y = 2x - \frac{4}{3}$	
17. $2x + y = 1$	
18. $3x - y = 1$	
19. $3x - y = 0$	
20. $2x + 3y = 3$	
21. $3x - 4y = -20$	
22. $4x + 10y = 5$	
23. $y = 3x + 5$	
24. $y = -2x - 2$	
25. $y = \frac{1}{3}x + \frac{5}{3}$	
26. $y = -\frac{2}{5}x - \frac{14}{5}$	
27. $y = \frac{1}{4}x - \frac{4}{3}$	
28. $y = \frac{1}{2}x + \frac{7}{12}$	
29. $3x - y = -5$	
30. $2x + y = -2$	
31. $x - 3y = -5$	
32. $2x + 5y = -14$	
33. $3x - 12y = 16$	
34. $6x - 12y = -7$	
35. $y = 2x + 2$	
36. $y = \frac{1}{2}x - 3$	
37. $y = 3$	
38. $y = -\frac{2}{3}x - 2$	
39. $y = -\frac{3}{4}x - \frac{1}{2}$	
40. $y = 0.4x + 2.3$	
41. See Website	

Extra Work Space