Section 3.1b – Special Cases of Linear Equations

This booklet belongs to:_____**Block:** _____

Horizontal Lines

- A horizontal line can be thought of as **all the points** on the graph where y has the same value
- The $\boldsymbol{slope}$ of a horizontal line $\boldsymbol{is}$ $\mathbf{0}$ (The rise is 0).
- Using a slope of 0, the slope intercept equation of a line is:

$$y = mx + b \quad \rightarrow \quad y = 0(x) + b \quad \rightarrow \quad \boldsymbol{y = b}$$

Equation of a Horizontal Line with $\boldsymbol{y}$– $\boldsymbol{intercept}$ $(\mathbf{0}, \boldsymbol{b})$

$$\boldsymbol{y = b}$$

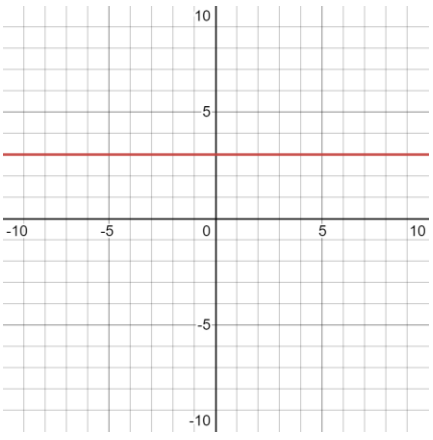
Vertical Lines

- A vertical line can be thought of as **all the points** on the graph where x has the same value
- The $\boldsymbol{slope}$ of a vertical line **is** $\boldsymbol{undefined}$ (The run is 0).
- The equation of a vertical line is $x = a$ by definition, since the slope is undefined.

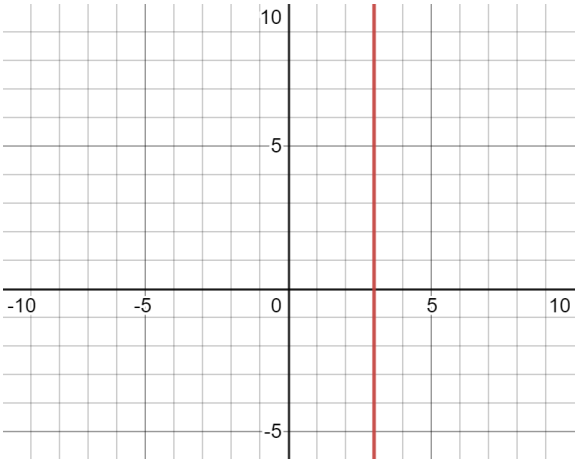
Equation of a Vertical Line with $\boldsymbol{x}$– $\boldsymbol{intercept}$ $(\boldsymbol{a}, \mathbf{0})$

$$\boldsymbol{x = a}$$

Example of Horizontal Line $\boldsymbol{y = 3}$



Example of Vertical Line $\boldsymbol{x = 3}$



Writing the Equation of a Line Through Two Points

- When **two points** are given, we now have the ability to write the equation of a line

Example 1: Write the equation of a line passing through $A(5, 2)$ and $B(1, -4)$ in $\boldsymbol{slope - intercept\ form}$.

Solution 1:

- First **find the slope** of the line. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{5 - 1} = \frac{6}{4} = \mathbf{\frac{3}{2}}$

- Now pick **either point** and **substitute it** into the $\boldsymbol{point - slope\ equation}$.

$y - y_1 = m(x - x_1) \rightarrow y - 2 = \frac{3}{2}(x - 5)$

$\rightarrow y - 2 = \frac{3}{2}x - \frac{15}{2}$

$\rightarrow y = \frac{3}{2}x - \frac{15}{2} + 2 \rightarrow y = \frac{3}{2}x - \frac{15}{2} + \frac{4}{2}$

$\rightarrow \boldsymbol{y = \frac{3}{2}x - \frac{11}{2}}$

- You get the same answer if you use the other point

$y - y_1 = m(x - x_1) \rightarrow y - (-4) = \frac{3}{2}(x - 1)$

$\rightarrow y + 4 = \frac{3}{2}x - \frac{3}{2}$

$\rightarrow y = \frac{3}{2}x - \frac{3}{2} - 4 \rightarrow y = \frac{3}{2}x - \frac{3}{2} - \frac{8}{2}$

$\rightarrow \boldsymbol{y = \frac{3}{2}x - \frac{11}{2}}$

- So, find the $\boldsymbol{Slope}$ first
- Then **substitute** in **either** one of the points for $(\boldsymbol{x_1, y_1})$
- Use algebra to get to $\boldsymbol{slope - intercept\ form}$
- You can then continue the algebra to get to $\boldsymbol{general\ form}$

Parallel and Perpendicular Lines

- Parallel lines have the $\boldsymbol{same\ slopes}$ but different $\boldsymbol{y-intercepts}$
- Perpendicular lines have slopes that are $\boldsymbol{negative\ reciprocals}$ of each other
- Knowing this we can determine if equations are **parallel, perpendicular of neither**

Example 2: In the following system of equations, determine if the lines are parallel, perpendicular or neither.

$$x + 2y = 6$$
$$-2x + y = 3$$

Solution 2:

- The short cut is remembering the slope of the **Standard Form** of a line, $Ax + By = C$, is: $-\frac{A}{B}$

$x + 2y = 6$ has a slope of $-\mathbf{\frac{1}{2}}$ $-2x + y = 3$ has a slope of $\mathbf{2}$

The **slopes** are **negative reciprocals** of each other, so the lines are **perpendicular**.

- I don't like relying on things to remember so it is important to be able to manipulate the equations using algebra to go from $\boldsymbol{Standard\ Form\ to\ Slope-Intercept\ Form}$

Example 3: In the following system of equations, determine if the lines are parallel, perpendicular, or neither.

$$3x - y = 5$$
$$-6x + 2y = 12$$

Solution 3:

- Put both equations into $\boldsymbol{Slope-intercept\ Form}$

$3x - y = 5 \quad \rightarrow \quad -y = -3x + 5 \quad \rightarrow \quad y = 3x - 5 \quad \rightarrow \quad \boldsymbol{m = 3}$

$-6x + 2y = 12 \quad \rightarrow \quad 2y = 6x + 12 \quad \rightarrow \quad y = 3x + 6 \quad \rightarrow \quad \boldsymbol{m = 3}$

The **slopes** are **equal**, so the lines are **parallel**.

Example 4: In the following system of equations, determine if the lines are parallel, perpendicular, or neither.

$$4x + 3y = 7$$
$$2x - 4y = 4$$

Solution 4:

- Using the **Standard Form** shortcut (Slope is $-\frac{A}{B}$):
  - $4x + 3y = 7$ has a slope of $-\frac{4}{3}$
  - $2x - y = 4$ has a slope of 2

- Changing the system of equations to **Slope-intercept form**:

$4x + 3y = 7 \rightarrow \quad 3y = -4x + 7 \quad \rightarrow \quad y = -\frac{4}{3}x + \frac{7}{3} \rightarrow \quad \boldsymbol{m = -\frac{4}{3}}$

$2x - y = 4 \quad \rightarrow \quad -y = -2x + 4 \quad \rightarrow \quad y = 2x - 4 \quad \rightarrow \quad \boldsymbol{m = 2}$

Both methods produce the **same result**.

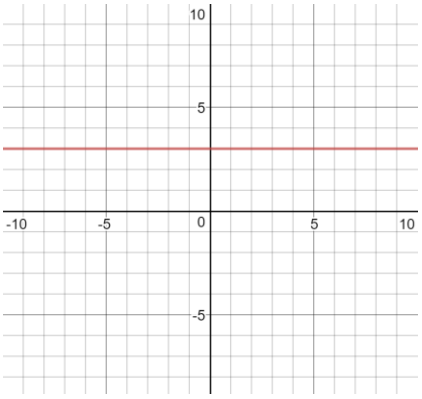
The **slopes** aren't the **same**, **or negative reciprocals of one another**, so the **lines** are **neither parallel nor perpendicular**.

Section 3.1b – Practice Problems

EMERGING LEVEL QUESTIONS

Determine the equation of the graph and explain why.

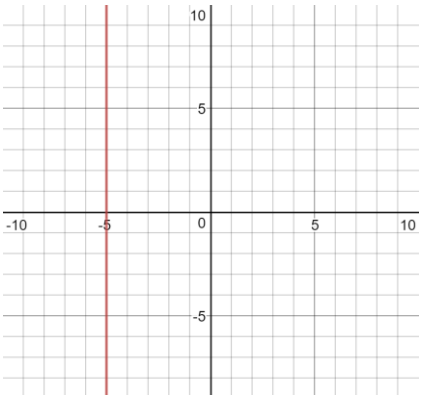
1.



Equation:

Why:

2.



Equation:

Why:

Determine the equation of a line through the given pair of points.

3. $(-4, 1) and (6, 1)$

4. $(1, -4) and (1, 6)$

5. $(-2, 0) and (5, 0)$

6. $(0, -2) and (0, 5)$

7. $(a, b) and (c, b)$

8. $(b, a) and (b, c)$

Write the equation of the line with the given information

9. $vertical, passes\ through\ (3, 6)$

10. $vertical, passes\ through\ (-2, -4)$

11. $horizontal, passes\ through\ (3, 6)$

12. $horinzontal, passes\ through\ (-2, -4)$

PROFICIENT LEVEL QUESTIONS

For each pair of equations, determine whether they are parallel, perpendicular, or neither

13. $2x + 5y = 7 \quad and \quad 4x + 10y = 2$

14. $-4x + 3y = 7 \quad and \quad -8x + 6y = 0$

15. $4x - 3y = 6 \quad and \quad 4x + 6y = -3$

16. $3x - 5y = 4 \quad and \quad 5x - 3y = 4$

17. $4x - 3y = 5 \quad and \quad 3x + 4y = 2$

18. $2x - 5y = -3 \quad and \quad 10x + 4y = 1$

19. $4x - y = 3 \quad and \quad x - 4y = -2$

20. $5x - 2y = 7 \quad and \quad 2x + 5y = 7$

Write the equation of a line passing though the given set of points in $\boldsymbol{slope - intercept\ form}$

21. $(3,5) and (2,4)$

22. $(5,-2) and (-3,1)$

23. $(-4,1) and (-2,-3)$

24. $(-1,-2) and (-6,-4)$

25. $(6,-2) and (-3,2)$

26. $(0,0) and (-3,2)$

EXTENDING LEVEL QUESTIONS

With the information provided, use reasoning to answer the following questions

27. If a line is horizontal, what is the slope of any line perpendicular to it?

28. If the graph of a linear equation has one point that is both the $x - intercept$ and $y - intercept$, where is that point?

29. What is the equation of the $x - axis$?

30. What is the equation of the $y - axis$?

31. What is the $x - intercept$ of the line $ax + by = c$?

32. What is the $slope$ of the line $ax + by = c$?

Section 3.1b – Answer Key

1. $y = 3$
2. $x = -5$
3. $y = 1$
4. $x = 1$
5. $y = 0$
6. $x = 0$
7. $y = b$
8. $x = b$
9. $x = 3$
10. $x = -2$
11. $y = 6$
12. $y = -4$
13. Parallel
14. Parallel
15. Neither
16. Neither
17. Perpendicular
18. Perpendicular
19. Neither
20. Perpendicular
21. $y = x + 2$
22. $y = -\frac{3}{8}x - \frac{1}{8}$
23. $y = -2x - 7$
24. $y = \frac{2}{5}x - \frac{8}{5}$
25. $y = -\frac{4}{9}x + \frac{2}{3}$
26. $y = -\frac{2}{3}x$
27. Undefined
28. $(0, 0)$
29. $y = 0$
30. $x = 0$
31. $x = \frac{c}{a}$
32. $-\frac{a}{b}$

Extra Work Space