Section 4.1a – Solving Linear Systems by Graphing

This booklet belongs to:_____________________________Block: ______

- If you think back to Grade 9, we talked about SOLUTIONS to equations of lines
- These were the (x, y) coordinates that satisfied the Slope-Intercept or Standard Form Equation
- What it did was kept the equation equal when we plugged in the values, see below:

Standard Form

Example 1:

- Does the line, $3x - 2y = -6$ go through the point $(2, 6)$?

Solution 1:

- In other words,
- The $(2, 6)$ a solution to the equation $3x - 2y = -6$?
    - So, sub in **2 for x** and **6 for y**

$$3(2) - 2(6) = -6$$

$$6 - 12 = -6$$

$$-6 = -6$$

Yes, it is a solution; the **line goes through the point**!

Slope-Intercept Form

Example 2:

- Does the line $y = 2x + 5$ go through the point $(1,8)$?

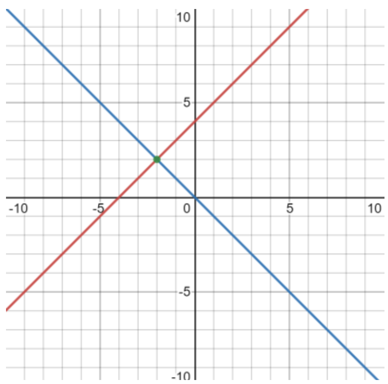
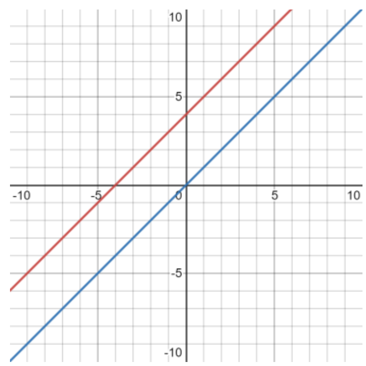
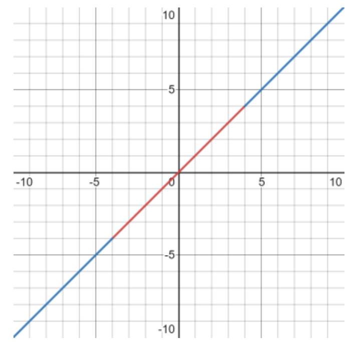
Solution 2:

- In other words,
- The $(1, 8)$ a solution to the equation $y = 2x + 5$
    - So, sub in 1 for x and 8 for y

$$8 = 2(1) + 5 \quad \rightarrow \quad 8 = 2 + 5 \quad \rightarrow \quad 8 = 7$$

Since $8 \neq 7$ it is NOT a solution; the **line goes DOES NOT go through the point**!

- So, the only difference in a **SYSTEM of EQUATIONS** is that we are looking for that (x, y) point that satisfies **2 or more equations** at the **same time**.
- We can do this in a number of different ways, graphing will be discussed first.
- Graphing a system of linear equations shows a **visual picture** of the problem **and a solution** to the system
- A linear system may have <u>**NO solutions, ONE solution, or INFINITE solutions**</u>

One Solution	**No Solutions**	**Infinite Solutions**
		
The graphs of two linear equations intersect at **one point**. There is **one solution** that is called a **consistent** system of **independent** equations.	The graphs of two linear equations are parallel lines. There is **no solution**, and the system is called **inconsistent**.	The graphs of the two linear equations are the same line. There are an **infinite number of solutions** to the system, because any ordered pair satisfies both equations. This is a **consistent** system of **dependent** equations.

<u>Solving a Linear System by Graphing</u>

1. Rewrite each equation in either **SLOPE-INTERCEPT OR STANDARD FORM**
2. **Graph** both equations on **the same grid**
3. Identify the **point of intersection** of the two graphs. The solution is **ordered pair** of the point of intersection.
4. **Check the solution** algebraically by **substituting** the ordered pair into each equation of the **original system**.
5. Label the point of intersection

Example 1: **Solve the System: $x + 2y = -4$ and $x - y = 5$ by graphing.**

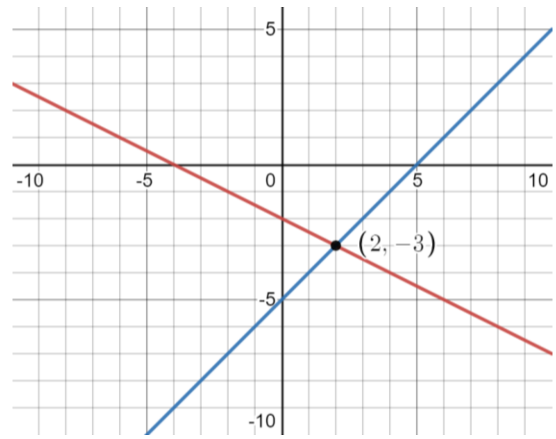
Solution 1:

$x + 2y = -4$

x	y
0	-2
-4	0
4	-4

$x - y = 5$

x	y
0	-5
5	0
2	-3



Graphing the two lines shows an intersection at $(2, -3)$

Therefore, the solution to the system is $(2, -3)$

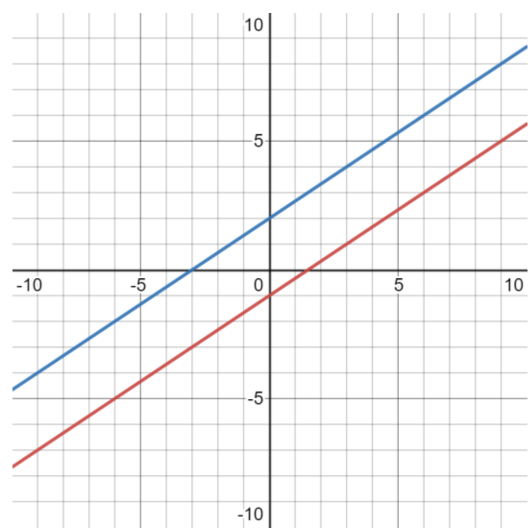
Example 2: **Solve the system: $2x - 3y = 3$ and $-2x + 3y = 6$ by graphing**

Solution 2: Let's look at graphing by changing the equations to Slope-Intercept Form

$$\mathbf{2x - 3y = 3}$$
$$-3y = -2x + 3$$
$$y = \frac{-2}{-3}x + \frac{3}{-3}$$
$$y = \frac{2}{3}x - 1$$
$$\boldsymbol{m = \frac{2}{3}}$$

$$-2x + 3y = 6$$
$$3y = 2x + 6$$
$$y = \frac{2}{3}x + \frac{6}{3}$$
$$y = \frac{2}{3}x + 2$$
$$\boldsymbol{m = \frac{2}{3}}$$

Same Slope so either the **SAME LINE** or **PARALLEL**



Graphing the two lines shows **no intersection point.**

Therefore, there is **no solution** to the system of equations.

Example 3: **Solve the system: $2x - y = 4$ and $-4x + 2y = -8$ by graphing**

Solution 3:

$2x - y = 4$

$m = 2$

x	y
0	-4
2	0
-1	-6

$$\frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{0-(-4)}{2-0}$$

$$\frac{4}{2}$$

$$m = 2$$

$-4x + 2y = -8$

$m = 2$

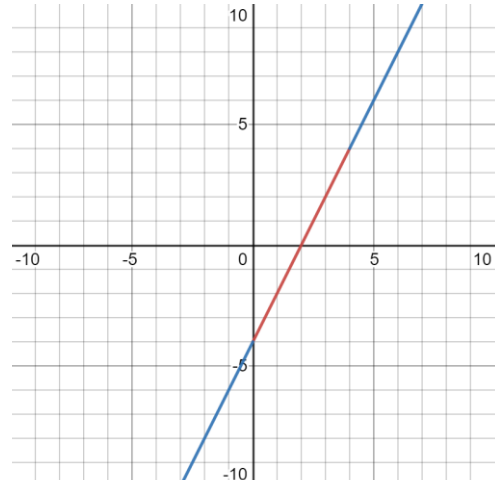
x	y
0	-4
2	0
-1	-6

$$\frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{0-(-4)}{2-0}$$

$$\frac{4}{2}$$

$$m = 2$$



Graphing the two lines shows **infinite** intersection points.

Therefore, there are an **infinite number of solutions** to the system of equations.

You may need to use all of your tools to accurately graph your equations. Perfect Intersection points are key.

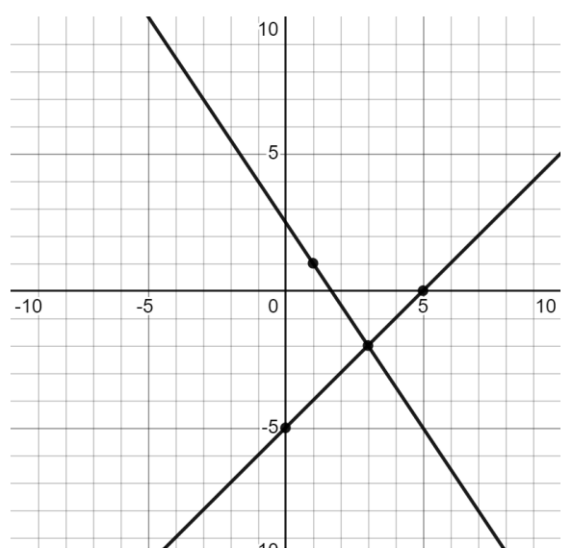
Graph: $x - y = 5$ and $3x + 2y = 5$

This equation is easy. We can find the x-int and y-int quite easily.

Slope intercept form gives us: $y = -\frac{3}{2}x + \frac{5}{2}$

We get the slope! But the y-intercept is an estimate. So, can we find a point?

x-intercept is also an estimate, so we have to look a little closer. Can you see a point that satisfies the equation? $\rightarrow$ $(1, 1)$



Section 4.1a – Practice Problems

EMERGING LEVEL QUESTIONS

Determine whether the ordered pairs are a solution to the linear system

1. $3x + y = 17 \ \ and \ \ 2x + 3y = 17; \ \ (5, 2)$

2. $2x + y = 11 \ \ and \ \ 3x + 2y = 19; \ \ (3, 5)$

3. $x + 2y = -2 \ \ and \ \ 2x + 5y = 23; \ \ (2, -4)$

4. $4x = 72 - y \ \ and \ \ 3x = -7y - 4; \ \ (6, -2)$

5. $-2y = x + 10 \ \ and \ \ 3x = 6y - 6; \ \ (-6, -2)$

6. $x = 2 \ \ and \ \ y = 3; \ \ (3, 2)$

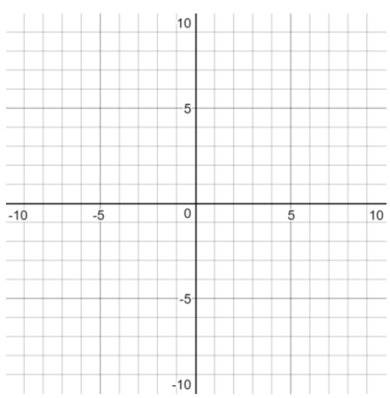
PROFICIENT LEVEL QUESTIONS

7. $\frac{1}{2}x + \frac{1}{3}y = 4 \ and \ \frac{1}{4}x + \frac{1}{3}y = 3; \quad (4, 6)$

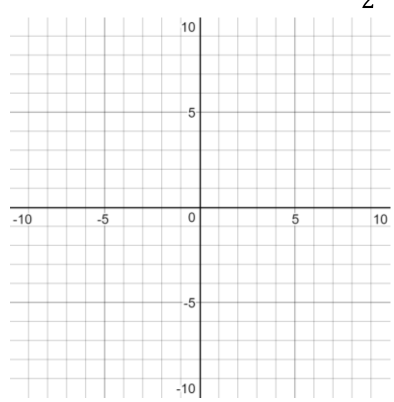
8. $0.3x - 0.2y = 4 \ and \ 0.2x + 0.3y = 1; \quad \left(\frac{140}{13}, \frac{-50}{13}\right)$

Solve by graphing

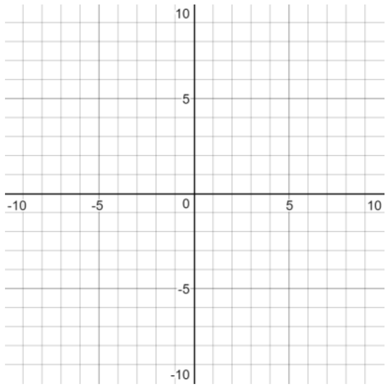
9. $2x - y = 3 \ and \ x + y = 3$



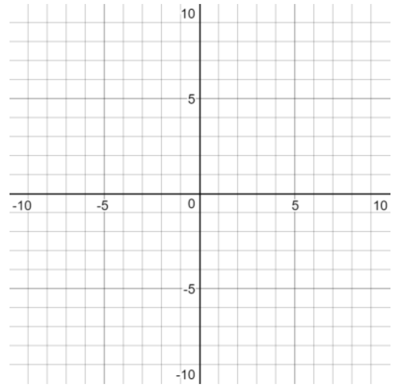
10. $x + 2y = -4 \ and \ y = -\frac{1}{2}x + 1$



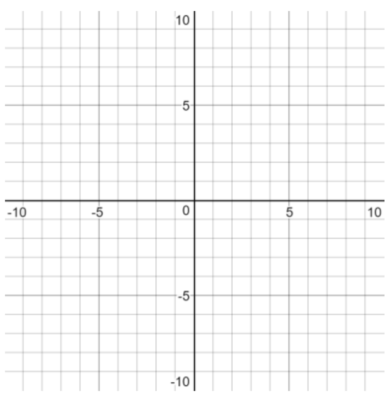
11. $f(x) = 2x - 4 \text{ and } 2x - y = 4$



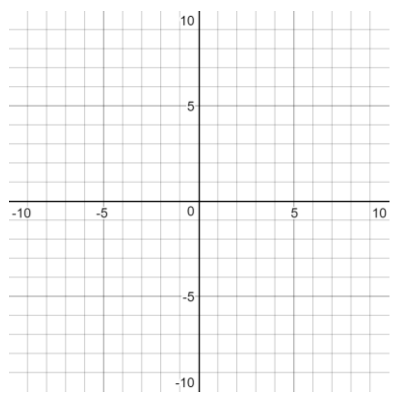
12. $x + y = -5 \text{ and } -2x + 1 = 1$



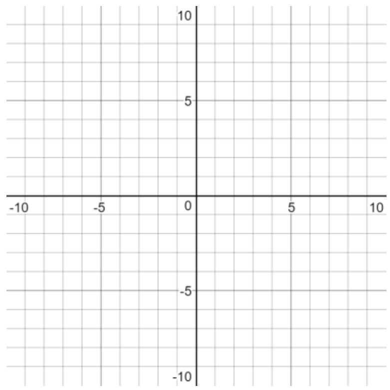
13. $2x - 3y = -1 \text{ and } 4x - y = 3$



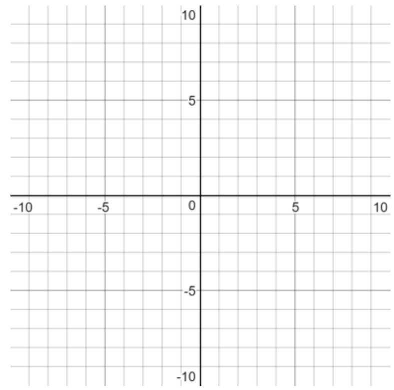
14. $x - \frac{y}{2} = -3 \text{ and } \frac{x}{3} - y = -1$



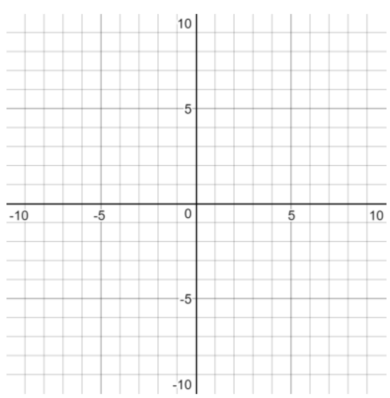
15. $x = 4 \ and \ 3x - 2y = 6$



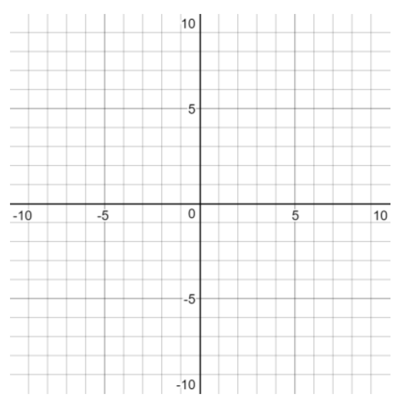
16. $2x - \frac{3}{2}y = 1 \ and \ f(x) = -2$



17. $f(x) = -x - 1 \ and \ 4x - 3y = 17$



18. $a - 2b = -8 \ and \ b - a = 6$



EXTENDING LEVEL QUESTIONS

19. Write a system of equations with the given solution.

a) $(3, -2)$

b) No solution

c) Infinite Solution

Section 4.1a – Answer Key

1. No
2. Yes
3. No
4. No
5. Yes
6. No
7. Yes
8. Yes
9. $(2, 1)$
10. No Solution
11. Infinite
12. $(0, -5)$
13. $(1, 1)$
14. $(-3, 0)$
15. $(4, 3)$
16. $(-1, -2)$
17. $(2, -3)$
18. $(-4, 2)$
19. Answers Vary

    See Website

Extra Work Space