Section 4.1b – Solving Linear Systems Using Addition

This booklet belongs to:______________________________Block: ______

- Solving a system by **graphing is limited** by the accuracy of the graph
- When the intersection point **is not** an exact integer, it is **difficult to determine** the coordinates from the graph
- Solving a system algebraically **does not depend on a graph**, and always gives **exact coordinates**
- The method used to solve a system algebraically is an extension of the **addition property** used to **solve an equation** for a single variable

Rule of Addition

For all real numbers $a, b, c \text{ and } d$; if $a = b$, and $c = d$, then $a + c = b + d$

Solving a Linear System by the Addition Method

1. Write the equations of the system in **STANDARD FORM:** $\boldsymbol{Ax + By = C}$
2. **Multiply** the terms of one equation, **or both** of the equations, by a **constant** such that the coefficients of $x \text{ or } y$ are **different only in their sign**
3. **Add the equations**, and solve the resulting equation
4. **Substitute** the value obtained in step 3 into **either** of the **original equations**, and solve for the remaining variable
5. Steps 3 and 4 give the solution to the system
6. To check the solution, take the values from steps 3 and 4, and substitute them into the **equation not used** in step 4.

Example 1: **Solve:** $\boldsymbol{x + y = 6}$ **and** $\boldsymbol{x - y = 4}$

Solution 1:

$$\begin{array}{l} x + y = 6 \\ \underline{x - y = 4} \end{array}$$

$2x = 10 \rightarrow x = 5$

To find y, substitute $x = 5$ in one of the original equations.

$x + y = 6 \rightarrow 5 + y = 6 \rightarrow y = 1$

Check: Substitute $(5, 1)$ into the equation not used, $x - y = 4 \rightarrow 5 - 1 = 4$ **True!**

The solution to the system is $(\mathbf{5}, \mathbf{1})$

Example 2: **Solve:** $\boldsymbol{y = x + 4}$ **and** $\boldsymbol{x + 2y = 5}$

Solution 2: Rewrite $y = x + 4$ as $-x + y = 4$

$$\begin{array}{l} -x + y = 4 \\ \underline{\ \ x + 2y = 5} \end{array}$$

$3y = 9 \rightarrow y = 3$

To find x, substitute $y = 3$ in one of the original equations.

$x + 2y = 5 \rightarrow x + 2(3) = 5 \rightarrow x = -1$

Check: Substitute $(-1, 3)$ into the equation not used,

$y = x + 4 \rightarrow 3 = -1 + 4$ **True!**

The solution to the system is $(-\mathbf{1}, \mathbf{3})$

Example 3: **Solve:** $\mathbf{2x - 3y = 2}$ **and** $\mathbf{x + 2y = 8}$

Solution 3: To obtain coefficients for x that differ only in sign**, multiply the second equation** by -2
Add the results to obtain an equation that has only one variable y.

$$\begin{aligned} 2x - 3y &= 2 \\ -2(x + 2y &= 8) \end{aligned} \quad \rightarrow \quad \begin{aligned} 2x - 3y &= 2 \\ \underline{-2x - 4y} &\underline{= -16} \\ -7y &= -14 \end{aligned} \quad \rightarrow \quad y = 2$$

To find x, substitute $y = 2$ in one of the original equations.

$$x + 2y = 8 \quad \rightarrow \quad x + 2(2) = 8 \quad \rightarrow \quad x = 4$$

Check: Substitute $(4, 2)$ in to $2x - 3y = 2 \quad \rightarrow \quad 2(4) - 3(2) = 2 \quad \rightarrow \quad 2 = 2$ **True!**

The solution to the system is $(\mathbf{4}, \mathbf{2})$

Example 4: **Solve:** $\mathbf{4x + 3y = 5}$ **and** $\mathbf{3x - 2y = 8}$

Solution 4: Multiply Equation 1 by 2 and Equation 2 by 3, then add the results

$$\begin{aligned} 2(4x + 3y &= 5) \\ 3(3x - 2y &= 8) \end{aligned} \quad \rightarrow \quad \begin{aligned} 8x + 6y &= 10 \\ \underline{9x - 6y} &\underline{= 24} \\ 17x &= 34 \end{aligned} \quad \rightarrow \quad x = 2$$

To find y, substitute $x = 2$ in one of the original equations.

$$4x + 3y = 5 \quad \rightarrow \quad 4(2) + 3y = 5 \quad \rightarrow \quad 3y = -3 \quad \rightarrow \quad y = -1$$

Check: Substitute $(2, -1)$ in to $3x - 2y = 8 \quad \rightarrow \quad 3(2) - 2(-1) = 8 \quad \rightarrow \quad 8 = 8$ **True!**

The solution to the system is $(\mathbf{2}, \mathbf{-1})$

Example 5: **Solve:** $\mathbf{3x - 2y = 1}$ **and** $\mathbf{-6x + 4y = 3}$

Solution 5: Multiply Equation one by 2, then add the results.

$$\begin{array}{lcr} 2(3x - 2y = 1) & & 6x - 4y = 2 \\ -6x + 4y = 3 & \rightarrow & \underline{-6x + 4y = 3} \\ & & 0 = 5 \end{array}$$

There is no solution so the lines are PARALLEL

Example 6: **Solve:** $\mathbf{2x + 5y = 2}$ **and** $\mathbf{-4x - 10y = -4}$

Solution 6: Multiply Equation one by 2, then add the results

$$\begin{array}{lcr} 2(2x + 5y = 2) & & 4x + 10y = 4 \\ \underline{-4x - 10y = -4} & \rightarrow & \underline{-4x - 10y = 4} \\ & & 0 = 0 \end{array}$$

There are infinite solutions so the lines are the SAME

Section 4.1b – Practice Problems

Solve by the Addition Method

1. $x - y = 4 \ \ and \ \ x + y = -6$

2. $x - 2y = -1 \ \ and \ \ -x + y = 1$

3. $x + 2y = 3 \ \ and \ \ x - y = 6$

4. $3x - 2y = -5 \ \ and \ \ 3x + y = -11$

PROFICENT LEVEL QUESTIONS

5. $x = 3y \ \ and \ \ y = 5x + 14$

6. $3x - 11 = 8y \ \ and \ \ x + 6y - 8 = 0$

7. $3x + 5y = 17 \ \ and \ \ 4x - y = -8$

8. $4x + 3y = 1 \ \ and \ \ 3x + 2y = 2$

9. $7x - 3y = -5 \ and \ 3x + 5y = -21$

10. $5x + 2y = 8 \ and \ 3x + 5y = 20$

11. $5x - 3y = 10.5 \ and \ 2x + 5y = -2$

12. $3x - 2y = 6 \ and \ -6x + 4y = -6$

13. $3x - 2y = 6 \ \ and \ -6x + 4y = -12$

14. $\frac{x}{3} + \frac{y}{4} = 1 \ \ and \ \frac{x}{2} - \frac{y}{8} = \frac{7}{2}$

EXTENDING LEVEL QUESTIONS

15. $\frac{3}{x} + \frac{4}{y} = \frac{5}{2} \ \ and \ -\frac{5}{x} + \frac{3}{y} = -\frac{7}{4}$

16. $\frac{6}{x} - \frac{9}{y} = 3 \ \ and \ \frac{5}{x} - \frac{6}{y} = 2$

17. $0.1x + 0.01y = 0.73 \ \ and \ \ 0.2x + 0.05y = 1.55$

18. $0.02x + \frac{y}{2} = 0.4 \ \ and \ \ \frac{x}{2} - 0.4y = -2.9$

19. $\frac{x}{2} + \frac{y}{5} = \frac{4}{5} \ \ and \ \ \frac{x}{6} - \frac{y}{2} = \frac{5}{6}$

20. $\frac{x}{4} - \frac{y}{2} = \frac{7}{24} \ \ and \ \ \frac{x}{3} + \frac{y}{2} = 0$

Section 4.1b – Answer Key

1. $(-1,-5)$
2. $(-1,0)$
3. $(5,-1)$
4. $(-3,-2)$
5. $(-3,-1)$
6. $\left(5,\frac{1}{2}\right)$
7. $(-1,4)$
8. $(4,-5)$
9. $(-2,-3)$
10. $(0,4)$
11. $\left(\frac{3}{2},-1\right)$
12. No Solution
13. Infinite Solutions
14. $(6,-4)$
15. $(2,4)$
16. No Solution
17. $(7,3)$
18. $(-5,1)$
19. $(2,-1)$
20. $\left(\frac{1}{2},-\frac{1}{3}\right)$

Extra Work Space