Section 4.2a – Solving Linear Systems by Substitution

This booklet belongs to:_____ Block: _____

- An algebraic method of substitution can be used to find the **exact solution** of a system
- The **substitution method** requires writing one of the systems in terms of x, or in terms of y

Solving a Linear System by the Substitution Method

1. Solve **one equation for one of its variables** in terms of the other variable; this becomes *equation* (3).

2. **Substitute** the equation form step 1 into the other equation, and solve that equation

3. Take the value solved for in step 2 and **substitute the value into *equation*** $(\mathbf{3})$

4. **Check** the solution by inserting the x and y values calculated in steps 2 and 3 into the equation not used in step 3.

Example 1: **Solve:** $\boldsymbol{2x + y = 4}$ **and** $\boldsymbol{3x + 4y = 1}$

Solution 1: Choose either equation and $solve\ for\ y$.

$$2x + y = 4 \quad \rightarrow \quad y = -2x + 4 \qquad Equation\ (3)$$

In the other equation, replace y with $(-2x + 4)$, and solve for x.

$$3x + 4y = 1 \quad \rightarrow \quad 3x + 4(-2x + 4) = 1$$
$$3x - 8x + 16 = 1$$
$$-5x = -15$$
$$x = 3$$

To find y, substitute $x = 3$ into $Equation\ (3)$.

$$y = -2x + 4 \quad \rightarrow \quad y = -2(3) + 4 \quad \rightarrow \quad y = -2$$

Check:

Substitute $(3, -2)$ in $3x + 4y = 1$ $\rightarrow$ $3(3) + 4(-2) = 1$ $\rightarrow$ $1 = 1$ $\boldsymbol{True}$!

The solution to the system is $(\mathbf{3}, -\mathbf{2})$

Example 2: **Solve:** $\mathbf{2x - 4y = 7}$ **and** $\mathbf{-x + 8y = -5}$

Solution 2: Choose either equation and solve for x.

$-x + 8y = -5 \quad \rightarrow \quad x = 8y + 5 \quad Equation\ (3)$

In the other equation, replace x with $(8y + 5)$, and solve for y.

$$2x - 4y = 7 \quad \rightarrow \quad 2(8y + 5) - 4y = 7$$
$$16y + 10 - 4y = 7$$
$$12y = -3$$
$$y = \frac{-3}{12} \quad \rightarrow \quad y = -\frac{1}{4}$$

To find *x*, substitute $y = -\frac{1}{4}$ into $Equation\ (3)$

$$x = 8y + 5 \quad \rightarrow \quad x = 8(-\frac{1}{4}) + 5 \quad \rightarrow \quad x = 3$$

Check:

Substitute $(3, -\frac{1}{4})$ in $2x - 4y = 7 \quad \rightarrow \quad 2(3) - 4(-\frac{1}{4}) = 7 \quad \rightarrow \quad 7 = 7$ **True!**

The solution to the system is $\left(\mathbf{3}, -\frac{\mathbf{1}}{\mathbf{4}}\right)$

Example 3: **Solve:** $\mathbf{3x - y = 5}$ **and** $\mathbf{-6x + 2y = -10}$

Solution 3: Choose either equation and solve for y.

$3x - y = 5 \quad y = 3x - 5 \quad Equation\ (3)$

In the other equation, replace y with $(3x - 5)$, and solve for x.

$$-6x + 2y = -10 \quad \rightarrow \quad -6x + 2(3x - 5) = -10$$
$$-6x + 6x - 10 = -10$$
$$\mathbf{0 = 0}$$

This is a true equation, therefore there are **infinite solutions**, and the lines must coincide.

Section 4.2a – Practice Problems

EMERGING LEVEL QUESTIONS

Solve by the substitution method

1. $y = -x + 2 \ and \ 2x - y = 4$

2. $x = 3y + 2 \ and \ x - 2y = 5$

3. $4x - 3y = 2 \ and \ y = 2x + 1$

4. $3x + 2y = 0 \ and \ x - 3y = 0$

PROFICIENT LEVEL QUESTIONS

5. $2x - y = 5 \ and \ -4x + 2y = -10$

6. $3x - y = 5 \ and \ -3x + y = 5$

7. $2x - 5y = 0 \ and \ x - y = 3$

8. $y = -3x - 8 \ and \ y = 15 - 2x$

9. $y = 3x + 4 \ \ and \ \ 2x - 3y = 2$

10. $y = -2x \ \ and \ \ x + 4y = 21$

11. $6x - y = 0 \ \ and \ \ 8x - 3y = 25$

12. $2s + t = -3 \ \ and \ 3s + 2t = -4$

13. $y = \frac{1}{3}x + 2 \ \ and \ 2x - 6y = -12$

14. $2x = 3y + 4 \ \ and \ \ 6x = 9y + 8$

EXTENDING LEVEL QUESTIONS

15. $\frac{1}{3}x - y = 3 \ and \quad 2x + \frac{1}{2}y = 5$

16. $\frac{x}{2} - \frac{2y}{3} = 2 \ and \ \frac{x}{4} + 3y = -4$

Solve the system of linear equations for k so that there are:

17. One Solution

$$y = 3x + 2 \ and \ y = kx + 2$$

18. No Solutions

$$y = 2x - 5 \ and \ 2x - y = k$$

Section 4.2a – Answer Key

1. $(2, 0)$
2. $(11, 3)$
3. $\left(-\frac{5}{2}, -4\right)$
4. $(0, 0)$
5. Infinite Solutions
6. No Solutions
7. $(5, 2)$
8. $(-23, 61)$
9. $(-2, -2)$
10. $(-3, 6)$
11. $(-2.5, -15)$
12. $(-2, 1)$
13. Infinite Solutions
14. No Solutions
15. $(3, -2)$
16. $\left(2, -\frac{3}{2}\right)$
17. $k \neq 3$
18. $k \neq 5$

Extra Work Space