Section 4.2b – Solving Problems with 2 Variables

This booklet belongs to:_____ Block: _____

Problem Solving with Two Variables

- Many students think this is a difficult topic, but with some helpful tips, students are capable of being very successful in solving these types of problems.

Steps to Follow in Problem Solving

1. **Read the problem very carefully** to determine what you are asked to solve.
2. Let x and y be unknown variables, and express **all** equations in terms of x and y.
3. Use an appropriate method, such as the **addition or substitution methods**, to solve for the unknown
4. **Check your answer** to make sure all conditions are satisfied in your problem.

Example 1: Adult tickets for the school play are $\$12.00$ and children's tickets are $\$8.00$. If the theater holds $300\ seats$ and the sold-out performance brings in $\$3280.00$, **how many** children and adults attended the play?

Solution 1: Let $x = number\ of\ adult\ tickets$, and $y = number\ of\ child's\ tickets$. **One equation** must deal with the **number of tickets**, and the **other equation** must deal with the **revenue from the tickets**.

	Adult Tickets	Children's Tickets	Total
No. of tickets	x	y	300
Revenue ($)	$12x$	$8y$	3280

Then $x + y = 300$ is the number of tickets sold
$12x + 8y = 3280$ is the revenue from the tickets

Using the **Substitution Method**:

$x + y = 300 \quad \rightarrow \quad y = 300 - x \quad Equation\ (3)$

$12x + 8y = 3280 \quad \rightarrow \quad 12x + 8(300 - x) = 3280 \quad \rightarrow \quad \boldsymbol{x = 220}$

- Now Solve for y

$y = 300 - x \quad \rightarrow \quad y = 300 - 220 \quad \rightarrow \quad \boldsymbol{y = 80}$

Check:

Substitute $(220, 80)$ in: $12x + 8y = 12(220) + 8(80) = 3280$

$3280 = 3280 \quad \boldsymbol{True}!$

Therefore 220 adult tickets and 80 children's tickets were sold.

Example 2: A small airplane makes a 2400km trip in 7.5 hours, and makes the **return trip** in 6 hours. If the plane travels at a **constant speed**, and the wind blows at a **constant rate**, find the airplane's airspeed, and the speed of the wind.

Solution 2: Let x = speed of the airplane with no wind
Let y = speed of the wind

	$Speed$ (km/h)	$Time$ (h)	$Distance$ (km)
With Wind	$x + y$	6	2400
Against Wind	$x - y$	7.5	2400

Then $x + y$ is the speed of the plane going **with the wind**
$x - y$ is the speed of the plane going **against the wind**

Using the **Addition Method**:

$distance = speed \cdot time$

$2400 = (x + y) \cdot 6$

$2400 = (x - y) \cdot 7.5$

$x + y = 400$

$\underline{x - y = 320}$

$2x = 720$

$\boldsymbol{x = 360}$ **km/h**

- Now Solve for y

$x + y = 400 \quad \rightarrow \quad 360 + y = 400 \quad \rightarrow \quad y = 40$ km/h

Check:

Substitute $(360, 40)$ in: $x - y = 320 \quad \rightarrow \quad 360 - 40 = 320 \quad \rightarrow \quad 320 = 320$ **True!**

The speed of the airplane is 360 km/h and the wind speed is 40 km/h

Example 3: A chemist has two acid solutions in stock: one is a 50% solution, and the other is a 80% solution. How much of each solution should be mixed to obtain 100 milliliters of a 68% solution?

Solution 3: Let x = number of milliliters of a 50% solution
Let y = number of milliliters of an 80% solution

	50% Solution	80% Solution	Total
$Acid\ (ml)$	x	y	100
$Strength$	$0.50x$	$0.80y$	$0.68 \cdot 100$

Then $x + y = 100$ is the total amount of acid

$0.50x + 0.80y = 0.68(100)$ is the strength of the acid

Using the **Substitution Method**:

$x + y = 100 \quad \rightarrow \quad y = 100 - x \quad Equation\ (3)$

$0.50x + 0.80y = 0.68(100) \quad \rightarrow \quad 0.50x + 0.80(100 - x) = 0.68(100)$

$0.50x + 80 - 0.80x = 68 \quad \rightarrow \quad \boldsymbol{x = 40}$ **mL**

- Now Solve for y

$y = 100 - x \quad \rightarrow \quad y = 100 - 40 \quad \rightarrow \quad y = 60$ mL

Check:

Substitute $(40, 60)$ in:

$0.50x + 0.80y = 68 \quad \rightarrow \quad 0.50(40) + 0.80(60) = 68 \quad \rightarrow \quad 68 = 68$ **True!**

Therefore, 40 milliliters of the 50% solution is needed, and 60 milliliters of the 80% solution is needed.

Section 4.2b – Practice Problems

PROFICENT LEVEL QUESTIONS

1. The sum of two numbers is six, and the difference is 10. Find the numbers.

2. The perimeter of a rectangle is 96cm. The length of the rectangle is 10cm more than the width. Find the length and width of the rectangle.

3. Two angles add up to $90°$. The larger angle is $12°$ more than five times the measure of the smaller angle. Find the measure of the two angles.

4. A cafeteria has a special: one hamburger and one order of fries is $\$5.49$, and one hamburger and two orders of fries is $\$6.99$. Determine the price of the hamburger.

5. Ellie is twice as old as her sister Kate. In seven years, the sum of their ages will be 20. How old are they now?

6. A car rental company charges x dollars to rent a truck, plus y dollars per kilometer. Find $x\ and\ y$ if the rental charge is $\$90$ for 100km, and $\$110$ for 150km.

EXTENDING LEVEL QUESTIONS

7. The cost of making pizzas includes a fixed cost, plus a cost per pizza. On Friday, 200 pizzas cost $\$800$ to make. On Saturday, 250 pizzas cost $\$950$ to make. Determine the daily cost, and the cost per pizza.

8. The sum of five times one number plus three times a second number is eight. The sum of three times the first number plus five times the second number is 24. Find the numbers.

9. A bottle of wine has 8% alcohol, another bottle has 15% alcohol. How much of each must be mixed to have 100 litres of 12.2% alcohol wine?

10. A plane travels 2835km with a tailwind, but only 1827km with a headwind in the same time. Find the speed of the plane and the speed of the wind, if time is 7 hours.

Section 4.2b – Answer Key

1. -2 and 8
2. 19 and 29
3. $13°$ and $77°$
4. $\$3.99$ and $\$1.50$
5. 2 and 4
6. $\$50$; $\$0.40$/km
7. $\$200$; $\$3$/pizza
8. -2 and 6
9. 40L of 8%
   60L of 15%
10. Plane: 333km
    Wind: 72km

Extra Work Space