

## Section 5.1a – Classifying and Simplifying Multiplication of Polynomials

This booklet belongs to: \_\_\_\_\_ Block: \_\_\_\_\_

### Term

- A number or a product of a number with one or more variables which can be raised to a power.

### Examples of Terms

Term	$5y$	$-2a^3$	$\frac{1}{2}x^2yz^4$	$x$	10
Coefficients	5	-2	$\frac{1}{2}$	1	10
Variables	$y$	$a$	$x, y, z$	$x$	<i>No variable</i>

### Polynomial

A term or sum of terms, in which all variables have **whole number exponents**, and in which **variables appear only in the numerator**.

### Examples and Non-Examples of Polynomials

Polynomial	Non-Polynomial
5	$x^{\frac{1}{2}}$
$\sqrt{2}x$	$2x + \sqrt{y}$
$3a^2 - 2a$	$\frac{1}{2x} + 4$
$\frac{3}{4}y + 3y^2 - 4$	$x^{-3} - 2x$

**Classifying Polynomials**

Monomial	A polynomial with one term	$3, 2x^2y, -3a$
Binomial	A Polynomial with two terms	$x + 2, 2x^2y + 3, x^2 - y$
Trinomial	A polynomial with three terms	$3x^2 + 2x - 3, \sqrt{2}x + y - z$
Polynomial	General term for expressions with more than three terms (can also be used to describe monomial, binomial, and trinomial)	$x^5 - 2x^4 + 3x^3 - 4x^2$

**Degree of a Polynomial**

The **degree** of a term in a polynomial is the **sum of the exponents** of the variables in the term.

The **degree of a polynomial** is the **term with the highest degree**.

**Examples:**

In  $3x^2 + 2x - 3$ , the term of the highest degree is  $3x^2$ , so the degree is 2.

In  $4x^2y^3 + z^4$ , the term of the highest degree is  $4x^2y^3$ , so the degree is 5. ( $2 + 3 = 5$ )

In  $-2x^2yz^2 + y^4$ , the term of the highest degree is  $-2x^2yz^2$ , so the degree is 5. ( $2 + 1 + 2 = 5$ )

**Leading Term**

The **term** with the **highest** degree.

**Consider:**  $3x^2y^3 - 2xy^2 + 2$

- The **degree** of the terms are: 5, 3, and 0 respectively
- This means that the **leading term** is the term with the **highest degree** so:  $3x^2y^3$
- It is good routine to always put polynomials in **descending order** with respect to **degree**

### Combining Like Terms

- Like terms are either **constant terms**, or **terms** that contain the **same variables** to the **same power**.

#### Example:

- $3x^2, 5x^2$  are like terms, they have the same variable and exponent
- $2xy^2, 3x^2y$  are not like terms because they have different exponents for each variable
- To **combine like terms**, **add or subtract** the **coefficients** of the terms.

**Example 1:** Simplify the expression  $4x^2 - 3y - x^2 + 5y$

**Solution 1:**  $4x^2$  and  $-x^2$  are like terms, so the coefficients can be added.  
 $-3y$  and  $5y$  are like terms, so the coefficients can be added.

**Therefore:**  $4x^2 - 3y - x^2 + 5y = 3x^2 + 2y$

### Evaluating Polynomials

- When a **constant** is **substituted** for a **variable** in a polynomial, the polynomial is evaluated for that constant.

#### Example:

When  $3x^2 - 2$  is evaluated for  $x = 4$ ,  
 the result is:  $3(4)^2 - 2 = 48 - 2 = 46$

When  $5x^3 + 6$  is evaluated for  $x = 2$ ,  
 the result is:  $5(2)^3 + 6 = 40 + 6 = 46$

When  $4xy^2 - 2x^2y$  is evaluated for  $x = 3$  and  $y = 4$ ,

the result is:  $4(3)(4)^2 - 2(3)^2(4) = 192 - 72 = 120$

### Multiplying a Monomial by a Monomial

- To multiply two **monomials** (one term), first multiply the constant factors, and then multiply the variable factors.

**Example 2:** Multiply

- $(2x^3)(-3x^4)$
- $(-3a^2b^3)(-2a^2b^5)$
- $(-3y^2z^3)(2yz^2)(4y^3z^2)$

**Solution 2:**

- $(2)(-3)(x^3)(x^4) = -6x^{3+4} = -6x^7$
- $(-3)(-2)(a^2)(a^2)(b^3)(b^5) = 6a^{2+2}b^{3+5} = 6a^4b^8$
- $(-3)(2)(4)(y^2)(y)(y^3)(z^3)(z^2)(z^2) = -24y^{2+1+3}z^{3+2+2} = -24y^6z^7$

### Multiplying a Monomial by a Binomial

- When multiplying a **monomial by a binomial**, use the **distributive property**
- DISTRIBUTIVE PROPERTY** or **WATERBOMB** is:  $a(b + c) = a * b + a * c$

**\*\*\*\*\*DON'T FORGET YOUR EXPONENT RULES\*\*\*\*\***

**Example 3:** Multiply

- $2(3 + 4)$
- $2x^2(3x^2 - 4y)$
- $-3y(x^4 + 2y^3)$

**Solution 3:**

- $2 \cdot 3 + 2 \cdot 4 = 6 + 8 = 14$
- $(2x^2)(3x^2) - (2x^2)(4y) = 6x^4 - 8x^2y$
- $(-3y)(x^4) + (-3y)(2y^3) = -3x^4y - 6y^4$

**Section 5.1a – Practice Problems**

**EMERGING LEVEL QUESTIONS**

For each polynomial, find the number of terms, the degree, and coefficients of each term

1. $3x^5$	2. $-2y^4$	3. $4x^3 - 2x^2$
4. $-3a^3 + 3a - 4^0$	5. $2x^3y^2 - 3x^2$	6. $2^3b^3 - 3^2$
7. $-x^3y^2z + \sqrt{2}xyz + 4z^3$	8. $\sqrt[3]{2}x^4y^3z^2p + p$	9. $x^3$

For each polynomial, simplify, then write the answer in descending order

10.  $3x^2 - 2x + 5x - x^2$

11.  $\frac{2}{3}x^4 + \frac{4}{3}x^4$

12.  $2.3x^2 + 3 - 4.1x^2 + 3x$

13.  $3y^4 - 2y^2 - y^4 - 2y$

14.  $-2x^3 - 2x^2 - 2x^3 + 2x^2$

15.  $x^2 - 2x + x^3 + x$

**PROFICIENT LEVEL QUESTIONS**

16.  $2x^2 - \frac{3}{4}x^3 + 6x^2 + \frac{2}{3}x^3$

17.  $-4y^3 - \frac{1}{2}y^5 + 5y^3 - \frac{1}{3}y^5$

18.  $\frac{1}{3}x - \frac{1}{2}x + x^2 + \frac{1}{3}x^2$

19.  $\sqrt{16}y - \frac{2}{3}y - y^3 - \frac{1}{4}y^2$

Find the value of the polynomial when  $x = -2$

20.  $-3x^2 + 2x - 1$

21.  $-3x^2 - 2x + 1$

22.  $2x^2 - 3x + 4$

23.  $-2x^2 - 3x - 4$

24.  $-x^4 + 2x^2 - 3$

25.  $x^4 - 2x^2 + 3$

26.  $-x^5 - 3x^3$

27.  $-x^4 - 3x^2$

Find each product.

28.  $3x^3(2x^4)$

29.  $-2a^2b^4(4ab^2)$

30.  $(3xy)(-4x^2y^2)$

31.  $(2ab)(-2ab)(2ab)$



32.  $(5x^3)(-2y^3)$

33.  $(-4a^4b^3)(2a^3b^2)(3ab)$

34.  $(a^2b^4)(a^3b)(-3b^2)$

35.  $(-r^4s^2t)(r^3st^2)(-rst)$

36.  $(-3ab^2)(2a^3b)(-a^2b^2)(-2a^3b^2)$

37.  $(-5a^3b^3c^2d^3)(-2ab^2cd^2)(-4a^2bc^3d)$

Find each product. Leave the answer in descending order.

38.  $3x(x - 4)$

39.  $-2x^2(x + 3)$

40.  $4y(-2y^2 + 3y)$

41.  $-5y(2y + 3y^2 - y^4)$

42.  $(3a^2)(2a)(-4 + 2a^2 - a^4)$

43.  $-2mn^4(-2mn + 3m^2n^2 - 4)$

**EXTENDING LEVEL QUESTIONS**

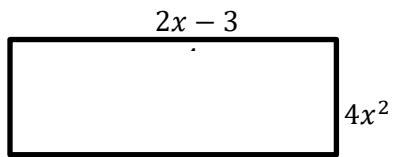
44.  $a^2bc(ab^2c^2 - a^2bc^2 - 2a^3b^3c)$

45.  $-abc^2(-a^2bc^3 + ab^2c - a^3c^2)$

46.  $(-x^2y)(xy^3)(xy - xy^2 + x^2y^2)$

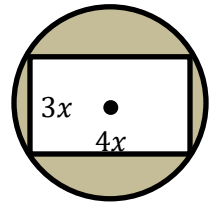
47.  $(-a^3b^2)(-a^2b)(-a^2b - a^2b^3 + a^3b)$

48. Determine the area



49. Determine the area of the shaded region in terms of  $x, y,$  and  $\pi$

Area of a Circle is:  
 $\pi r^2$



**Answer Key - Section 5.1a**

1.  
Terms: 1  
Coefficients: 3  
Degree: 5
2.  
Terms: 1  
Coefficients: -2  
Degree: 4
3.  
Terms: 2  
Coefficients: 4, -2  
Degree: 3, 2
4.  
Terms: 3  
Coefficients: -3, 3, -1  
Degree: 3, 1, 0
5.  
Terms: 2  
Coefficients: 2, -3  
Degree: 5, 2
6.  
Terms: 2  
Coefficients: 8, -9  
Degree: 3, 0
7.  
Terms: 3  
Coefficients: -1,  $\sqrt{2}$ , 4  
Degree: 6, 3, 3
8.  
Terms: 2  
Coefficients:  $\sqrt[3]{2}$ , 1  
Degree: 10, 1
9.  
Terms: 1  
Coefficients: 1  
Degree: 3
10.  $2x^2 + 3x$   
11.  $2x^4$   
12.  $-1.8x^2 + 3x + 3$
13.  $2y^4 - 2y^2 - 2y$   
14.  $-4x^3$   
15.  $x^3 + x^2 - x$   
16.  $-\frac{1}{12}x^3 + 8x^2$   
17.  $-\frac{5}{6}y^5 + y^3$   
18.  $\frac{4}{3}x^2 - \frac{1}{6}x$
19.  $-y^3 - \frac{1}{4}y^2 + \frac{10}{3}y$   
20. -17  
21. -7  
22. 18  
23. -6  
24. -11  
25. 11  
26. 56  
27. -28  
28.  $6x^7$   
29.  $-8a^3b^6$   
30.  $-12x^3y^3$   
31.  $-8a^3b^3$   
32.  $-10x^3y^3$   
33.  $-24a^8b^6$   
34.  $-3a^5b^7$   
35.  $r^8s^4t^4$   
36.  $-12a^9b^7$   
37.  $-40a^6b^6c^6d^6$   
38.  $3x^2 - 12x$   
39.  $-2x^3 - 6x^2$   
40.  $-8y^3 + 12y^2$   
41.  $5y^5 - 15y^3 - 10y^2$   
42.  $-6a^7 + 12a^5 - 24a^3$   
43.  $-6m^3n^6 + 4m^2n^5 + 8mn^4$   
44.  $-2a^5b^4c^2 - a^4b^2c^3 + a^3b^3c^3$   
45.  $a^3b^2c^5 + a^4bc^4 - a^2b^3c^3$   
46.  $-x^5y^6 + x^4y^6 - x^4y^5$   
47.  $-a^7b^6 + a^8b^4 - a^7b^4$   
48.  $8x^3 - 12x^2$   
49.  $6.25\pi x^2 - 12x^2$

**Extra Work Space**