Section 5.2a – Factoring Polynomials Part 1

This booklet belongs to:_____________________________Block: ______

Removing Common Factors

- To **factor** a polynomial is to express it as a **product (things MULTIPLYING together)**
- Removing a **common factor** is the most basic form of factoring
- If every term has at least one factor that is the same, it is known as the **common factor**
- Removing the **greatest common factor** is the best approach
- Factoring is another way of say

Example 1: Factor

a) $5x + 10$
b) $3x^2 - 6x$
c) $12x^4 - 8x^3 + 4x^2$

Solution 1:

a) $5x + 10 = 5(x) + 5(2) \quad = \mathbf{5(x + 2)}$

b) $3x^2 - 6x = (3x)(x) - 3x(2) \quad = \mathbf{3x(x - 2)}$

c) $12x^4 - 8x^3 + 4x^2 \quad = 4x^2(3x^2) - 4x^2(2x) + 4x^2(1)$

$= \mathbf{4x^2(3x^2 - 2x + 1)}$

- **NOTE:** If an entire term is the greatest common factor then a "1" must be used to hold its place (as seen in example c)

- **NOTE:** To make sure the solution is correct, you can check your answer by multiplying in

Example 2: Factor $x(x + 1) + 3(x + 1)$

Solution 2: A common factor does not have to be a monomial, $(x + 1)$ is common to both terms and can be taken out

$x(x + 1) + 3(x + 1) = \mathbf{(x + 1)(x + 3)}$

Factoring by Grouping

- When a polynomial has 4 terms, we can remove factors by **grouping**
- To factor by **grouping**
  - Group the polynomial into two pairs of two
  - Factor each pair of two
  - Remove the common factor

Example 3: Factor $x^3 + x^2 + 3x + 3$

Solution 3:

$$\begin{aligned} x^3 + x^2 + 3x + 3 &= (x^3 + x^2) + (3x + 3) \\ &= x^2(x + 1) + 3(x + 1) \\ &= \mathbf{(x + 1)(x^2 + 3)} \end{aligned}$$

Example 4: Factor $2x^3 - 6x^2 + x - 3$

Solution 4: This problem can be grouped in two ways

$$\begin{aligned} 2x^3 - 6x^2 + x - 3 &= (2x^3 - 6x^2) + (x - 3) \quad or \quad = (2x^3 + x) + (-6x^2 - 3) \\ &= 2x^2(x - 3) + (x - 3) \qquad\quad = x(2x^2 + 1) - 3(2x^2 + 1) \\ &= \mathbf{(x - 3)(2x^2 + 1)} \qquad\qquad = \mathbf{(x - 3)(2x^2 + 1)} \end{aligned}$$

Example 5: Factor $a^2 + ab - 2a - 2b$

Solution 5: This problem can be grouped in two ways

$$\begin{aligned} a^2 + ab - 2a - 2b &= (a^2 + ab) + (-2a - 2b) \quad or \quad = (a^2 - 2a) + (ab - 2b) \\ &= a(a + b) - 2(a + b) \qquad\qquad = a(a - 2) + b(a - 2) \\ &= \mathbf{(a + b)(a - 2)} \qquad\qquad\qquad = \mathbf{(a - 2)(a + b)} \end{aligned}$$

Note: Solutions in the previous examples are the same since $(\boldsymbol{m} \cdot \boldsymbol{n}) = (\boldsymbol{n} \cdot \boldsymbol{m})$

Section 5.2a – Practice Problems

EMERGING LEVEL QUESTIONS

Factor out the greatest common factor

1. $18x^3 - 27x$

2. $24a^3 + 18a$

3. $\frac{1}{3}y^2 - \frac{4}{3}y$

4. $3x^2 + 3x - 6$

5. $8b^2 - 4b + 20$

6. $5c^5 - 10c^3 + 15c$

7. $12x^3y + 6xy^2$

8. $6x^7 - 9x^6 - 57x^5 + 3x^4$

Factor

9. $x(2x+1)+3(2x+1)$

10. $x(x+1)+(x+1)$

11. $3y(3y+1)-2(3y+1)$

12. $6y^2(y-3)+5(y-3)$

13. $a(a+2b)+b(a+2b)$

14. $3c(1-2d)-2d(1-2d)$

15. $x(x-1)-3(1-x)$

16. $(x+1)(x+1)-3(x+1)$

PROFICIENT LEVEL QUESTIONS

Factor by grouping

17. $x^2 + xy + 2x + 2y$

18. $a^2 + ab - 2a - 2b$

19. $6y^2 + 12y - 3y - 6$

20. $a^2 - 5a + ab - 5b$

21. $x^2 + 2x - 2y - xy$

22. $1 + b - a - ab$

23. $2x^3 + 12x^2 - 5x - 30$

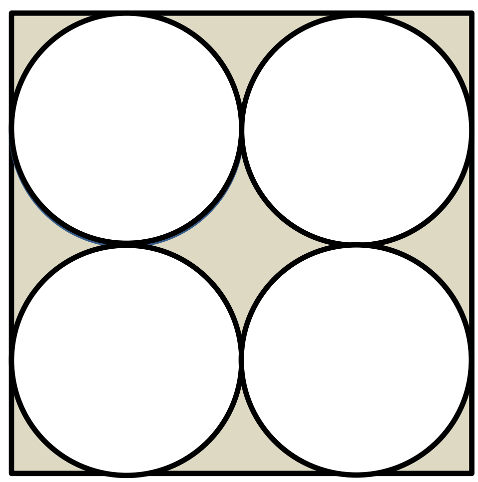
24. $x^3 + 5x^2 - 3x - 15$

25. $2x^3 - 6x^2 - 9x + 27$

26. $3b^3 + a^2b - 3a - ab^4$

EXTENDING LEVEL QUESTION

27. Determine the shaded area in factored form, if the circles are each of radius r.



Section 5.2a – Answer Key

1. $9x$
2. $6a$
3. $\frac{1}{3}y$
4. 3
5. 4
6. $5c$
7. $6xy$
8. $3x^4$
9. $(2x+1)(x+3)$
10. $(x+1)(x+1)$
11. $(3y+1)(3y-2)$
12. $(6y^2+5)(y-3)$
13. $(a+b)(a+2b)$
14. $(3c-2d)(1-2d)$
15. $(x+3)(x-1)$
16. $(x+1)(x-2)$
17. $(x+2)(x+y)$
18. $(a+b)(a-2)$
19. $3(2y-1)(y+2)$
20. $(a+b)(a-5)$
21. $(x-y)(x+2)$
22. $(1-a)(1+b)$
23. $(2x^2-5)(x+6)$
24. $(x^2-3)(x+5)$
25. $(2x^2-9)(x-3)$
26. $(3-ab)(b^3-a)$
27. $4r^2(4-\pi)$

Extra Work Space