

## Section 5.2b – Factoring Polynomials Part 2

This booklet belongs to: \_\_\_\_\_ Block: \_\_\_\_\_

### Factoring Quadratics (Polynomials of degree 2): $x^2 + bx + c$

Consider this:  $(x + a)(x + b) = x^2 + bx + ax + ab$

$$x^2 + (b + a)x + ab$$

- By looking at this we see that:
  - The **first term** is the **product** of  $x$  and  $x$
  - The **coefficient** of the **middle term** is the **sum** of  $a$  and  $b$
  - The **last term** is the **product** of  $a$  and  $b$
- This leads us to the **general rule**:

When factoring  $x^2 + bx + c$ , look for **two factors of  $c$** , that **multiply** to the **coefficient** of the **last term**, and **add** to the **coefficient** of the middle term.

**Example 1:** Factor  $x^2 + 7x + 12$

**Solution 1:** What two numbers **add to 7** and **multiply to 12**?

- Integers that multiply to 12: (1, 12) (2, 6) (3, 4) (-1, -12) (-2, -6) (-3, -4)
- Only integers +3 and +4 add to 7
- Therefore  $x^2 + 7x + 12 = (x + 3)(x + 4)$
- We can **check our answer** using **FOIL**:  $(x + 3)(x + 4)$

$$= x^2 + 3x + 4x + 12$$

$$= x^2 + 7x + 12$$

**Example 2:** Factor  $x^2 + 8 - 6x$

**Solution 2:** First **arrange** the polynomial in **descending order** of powers

- $x^2 + 8 - 6x = x^2 - 6x + 8$
- -4 and -2 add to -6 and multiply to +8
- Therefore:  $x^2 - 6x + 8 = (x - 4)(x - 2)$
- We can **check using FOIL**

**Example 3:** Factor  $5x^2 + 35x + 60$

**Solution 3:** Always look for a **common factor** first. The **largest common factor** is **5**

- Therefore:  $5x^2 + 35x + 60 = 5(x^2 + 7x + 12)$
- Now we can factor the Quadratic like we did previous:
- Two numbers that **multiply** to +12 and **add** to +7
- +4 and +3 get the job done
- So  $5(x^2 + 7x + 12) = 5(x + 4)(x + 3)$
- **Check** your answer **using FOIL**

**Example 4:** Factor  $-x^2 + 5x + 6$

**Solution 4:** First factor out  $-1$ , so that the **coefficient** of  $x^2$  becomes +1.

- So  $-x^2 + 5x + 6$  becomes  $-(x^2 - 5x - 6)$ , now factor  $(x^2 - 5x - 6)$
- $-6$  and  $1$  **multiply** to  $-6$  and **add** to  $-5$
- Therefore  $-x^2 + 5x + 6 = -(x^2 - 5x - 6) = -(x - 6)(x + 1)$
- **Note the factors are:  $(x - 6)(x + 1)$  and  $-1$**

**Example 5:** Factor  $-3x^4 - 18x^3 - 27x^2$

**Solution 5:** First look for a common factor. The largest here is  $-3x^2$ , factor it out

- So  $-3x^4 - 18x^3 - 27x^2$  becomes  $-3x^2(x^2 + 6x + 9)$ , now factor  $(x^2 + 6x + 9)$
- +3 and +3 **multiply** to +9 and **add** to +6
- Therefore  $-3x^4 - 18x^3 - 27x^2 = -3x^2(x^2 + 6x + 9) = -3x^2(x + 3)^2$
- **Note the factors are:  $(x + 3)(x + 3)$  and  $-3x^2$**

**SUMMARY OF FACTORING QUADRATICS**

1. Arrange the polynomial in **descending order** of powers
2. When:
  - The **last term** is **positive**, the **factors of  $c$**  are **both positive, or both negative**.
  - If the **middle term** is **positive**, **both integers** are **positive**.
  - If the **middle term** is **negative**, **both integers** are **negative**.

**Example:**  $x^2 + 7x + 12 = (x + 4)(x + 3)$

- The **last term** is **positive**, and the **middle term** is **positive**, therefore the **factors of 12** are **both positive**.
- **Opposite** if the **middle term** was **negative** and the **last positive**.

3. When:
  - The **last term** is **negative**, the **factors of  $c$**  have **opposite signs**.
  - The **larger numeric value** takes the **sign** of the coefficient of the **middle term**.

**Example:**  $x^2 - x - 6 = (x - 3)(x + 2)$

- The **last term** is **negative**, therefore the **signs of the factor** of 6 are **opposite of each other**
- Since the **middle term** is **negative** the **larger numeric value** has a **negative sign**.

**Example:**  $x^2 + 2x - 15 = (x + 5)(x - 3)$

- The **last term** is **negative**, therefore the **signs of the factor** of 15 are **opposite of each other**
- Since the **middle term** is **positive** the **larger numeric value** has a **positive sign**.

**Special Factors**

For trinomial  $ax^2 + bx + c$  to be a perfect square:

- a) The **last term** must be a **positive**, and a **perfect square**
- b) The **first term** must be a **perfect square**
- c) The **coefficient of the middle term** is the **square root of the first term multiplied by the square root of the coefficient of the last term**, then **doubled**.

**Example:**  $x^2 + 8x + 16 = (x + 4)^2$

**Example:**  $x^2 - 8x + 16 = (x - 4)^2$

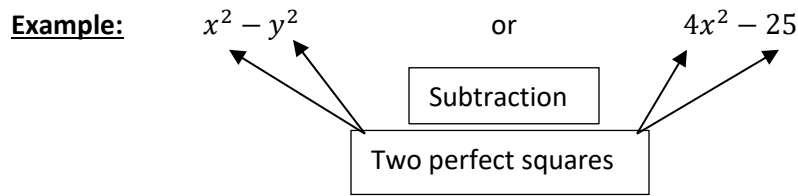
**Factoring Perfect Square Trinomials**

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

**Difference of Squares**

- Whenever we see **subtraction**, **two square terms** only, and **no term with degree one**, we have the possibility of a **difference of squares**



**Difference of Squares**

$$a^2 - b^2 = (a + b)(a - b)$$

**Example:**

$$4x^2 - 25 = (2x + 5)(2x - 5)$$

Check using FOIL  
The middle (degree 1 term), cancels out!

**Section 5.2b – Practice Problems****EMERGING LEVEL QUESTIONS**

Give four examples for  $b$  so that the following trinomials can be factored

1.  $x^2 + bx + 6$

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2.  $x^2 + bx + 4$

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3.  $x^2 + bx - 8$

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4.  $x^2 + bx - 6$

**PROFICIENT LEVEL QUESTIONS**

Give positive and negative examples for  $c$  so that the following trinomials can be factored

5.  $x^2 + 6x + c$

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6.  $x^2 - 4x + c$

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7.  $x^2 + x + c$

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8.  $x^2 - 5x + c$

9. A student factored  $x^3 - 5x^2 - 14x$  into  $(x - 7)(x + 2)$ . Explain the error that was made.

**EMERGING LEVEL QUESTIONS**

Factor

10.  $a^2 + 9a + 8$

11.  $b^2 + 16b + 15$

12.  $c^2 + 10c + 24$

13.  $d^2 + 7d + 10$

14.  $x^2 - 18x + 72$

15.  $y^2 - 20y + 91$

16.  $z^2 - 13z + 36$

17.  $u^2 - 4u + 4$

18.  $l^2 + 7l - 30$

19.  $m^2 + 4m - 12$

**PROFICIENT LEVEL QUESTIONS**

Factor Completely

20.  $3x^2 + 15x + 12$

21.  $4y^2 + 20y + 24$

22.  $-5x^2 + 25x - 20$

23.  $-2y^2 + 58y - 200$

24.  $-x^2 - 6x + 27$

25.  $-x^2 + 7x + 44$

26.  $x^3 + 8x^2 - 20x$

27.  $-2x^4 - 4x^3 + 30x^2$

28.  $-x^3y - x^2y^2 + 6xy^3$

29.  $2x^4 - 16x^3y + 32x^2y^2$

30.  $-x^3y^2 - 3x^2y^3 + 4xy^4$

31.  $x^6 - 11x^5y + 28x^4y^2$

**EXTENDING LEVEL QUESTIONS**

Factor Completely

32.  $(2a + 5)y^2 + 9(2a + 5)y - 10(2a + 5)$

33.  $x^3(a + b) - 6x^2(a + b) + 8x(a + b)$

34.  $(2a + b)x^2 - 12(2a + b)x + 27(2a + b)$

35.  $(3a - b)y^2 - 13(3a - b)y + 40(3a - b)$



36.  $x^4 + x^2 + 1$  (Very Challenging)

37.  $(2x + 3)^2 + (2xz + 3z) - 20z^2$

38.  $(x - 2y)^2 - 8a(x - 2y) + 15a^2$

39.  $(5x - y)^2 + (10xz - 2yz) - 24z^2$

40. The volume of a rectangular solid is  $(x^3 + 7x^2 + 12x)cm^3$ . Determine its dimensions in terms of  $x$ .

41. A sheet of cardboard measuring  $5in$  by  $7in$  has squares  $x$  inches wide cut from each corner. Then the sides are folded up to form an open top box. Express the volume of the box in factored form.

**PROFICIENT LEVEL QUESTIONS**

Factor each binomial completely

42.  $x^2 - 1$

43.  $4x^2 - 1$

44.  $y^2 - 25$

45.  $25y^2 - 9$

46.  $4 - 9z^2$

47.  $16 - 25y^2$

48.  $16x^2 - 9y^2$

49.  $25x^4 - 81y^6$

50.  $16x^2y^8 - 4$

51.  $20x^2 - 5y^2$

52.  $(x + 1)^2 - y^2$

53.  $4 - (x + 2)^2$

Factor each perfect square trinomial completely

54.  $x^2 + 10x + 25$

55.  $x^2 + 8x + 16$

56.  $y^2 - 12y + 36$

57.  $y^4 - 6y^2 + 9$

58.  $2z^2 - 28z + 98$

59.  $-9x^2 - 24xy - 16y^2$

## Section 5.2b – Answer Key

1. 7, 5, -7, -5
2. 5, -5, 4, -4
3. 7, -7, 2, -2
4. -5, 5, -1, 1
5. 5, 8, -7, -16
6. 4, 3, -5, -12
7.  $\frac{1}{4}, \frac{3}{16}, -2, -6$
8. 6, 4, -6, -14
9.  $x(x - 7)(x + 2)$
10.  $(a + 1)(a + 8)$
11.  $(b + 15)(b + 1)$
12.  $(c + 4)(c + 6)$
13.  $(d + 2)(d + 5)$
14.  $(x - 12)(x - 6)$
15.  $(y - 7)(y - 13)$
16.  $(z - 9)(z - 4)$
17.  $(u - 2)(u - 2)$
18.  $(l + 10)(l - 3)$
19.  $(m + 6)(m - 2)$
20.  $3(x + 1)(x + 4)$
21.  $4(y + 2)(y + 3)$
22.  $-5(x - 4)(x - 1)$
23.  $-2(y - 25)(y - 4)$
24.  $-(x + 9)(x - 3)$
25.  $-(x - 11)(x + 4)$
26.  $x(x + 10)(x - 2)$
27.  $-2x^2(x + 5)(x - 3)$
28.  $-xy(x + 3y)(x - 2y)$
29.  $2x^2(x - 4y)(x - 4y)$
30.  $-xy^2(x + 4y)(x - y)$
31.  $x^4(x - 7y)(x - 4y)$
32.  $(2a + 5)(y + 10)(y - 1)$
33.  $x(a + b)(x - 4)(x - 2)$
34.  $(2a + b)(x - 9)(x - 3)$
35.  $(3a - b)(y - 8)(y - 5)$
36.  $(x^2 + 1 - x)(x^2 + 1 + x)$
37.  $(2x + 3 + 5z)(2x + 3 - 4z)$
38.  $(x - 2y - 3a)(x - 2y - 5a)$
39.  $(5x - y + 6z)(5x - y - 4z)$
40.  $x(x + 3)(x + 4)$
41.  $x(7 - 2x)(5 - 2x)$
42.  $(x + 1)(x - 1)$
43.  $(2x + 1)(2x - 1)$
44.  $(y + 5)(y - 5)$
45.  $(5y + 3)(5y - 3)$
46.  $(2 - 3z)(2 + 3z)$
47.  $(4 - 5y)(4 + 5y)$
48.  $(4x - 3y)(4x + 3y)$

49.  $(5x^2 - 9y^3)(5x^2 + 9y^3)$
50.  $4(2xy^4 - 1)(2xy^4 + 1)$
51.  $5(2x - y)(2x + y)$
52.  $(x + 1 - y)(x + 1 + y)$
53.  $(x + 4)(-x)$
54.  $(x + 5)^2$
55.  $(x + 4)^2$
56.  $(x - 6)^2$
57.  $(y^2 - 3)^2$
58.  $2(z - 7)^2$
59.  $-(3x + 4y)^2$

**Extra Work Space**