

Section 5.3 – Factoring Polynomials Part 3

This booklet belongs to: _____ Block: _____

Factoring Quadratics: $ax^2 + bx + c$

In this section a will have an integer value greater than 1

I’m going to show you two methods; it’s called the **Factor by Grouping and The AC Method**.

The AC Method

The “AC” Method (Factoring Trinomials)

- The “AC” method is a technique used to factor trinomials
- A trinomial consists of three terms ($ax^2 + bx + c$).

Example of “AC” method:

1. $ax^2 + bx + c$
 $6x^2 + 7x + 2$

2. Multiply the **coefficient of the a term** with the **coefficient of the c term** and rewrite the polynomial with the starting term now x^2

So $6x^2 + 7x + 2$ becomes $x^2 + 7x + 12$

3. a) Find two numbers **that multiply** to +12 and **add** to +7

These are: $+ 3$ and $+ 4$

b) Rewrite the **factored form** of the new version of the Polynomial

$$(x + 3)(x + 4)$$

c) Now **divide the two factors** by the **original a term** and **simplify the fractions**

$$\left(x + \frac{3}{6}\right)\left(x + \frac{4}{6}\right) \rightarrow \left(x + \frac{1}{2}\right)\left(x + \frac{2}{3}\right)$$

d) If you are **not left with a denominator** then you are **done**. If a **denominator remains, rewrite it in front** of the x **term**.

Can’t simplify
so write the
2 in front of
the x

$$\left(x + \frac{1}{2}\right)\left(x + \frac{2}{3}\right)$$

Can’t simplify
so write the
3 in front of
the x

4. Rewrite as the factored form: $(2x + 1)(3x + 2)$ This is the Factored Form.

5. **Check using FOIL**

Example 1: Factor $2x^2 + 7x - 4$

Solution 1: $2x^2 + 7x - 4$ is first changed to $x^2 + 7x - 8$

$$(x + 8)(x - 1) \quad \text{Factor}$$

$$\left(x + \frac{8}{2}\right)\left(x - \frac{1}{2}\right) \quad \text{Divide each constant by 2, the original coefficient of } 2x^2$$

$$(x + 4)\left(x - \frac{1}{2}\right) \quad \text{Simplify}$$

$$(x + 4)(2x - 1) \quad \text{If there is fraction left after division, it becomes the coefficient of } x$$

Check using FOIL: $(x + 4)(2x - 1) = 2x^2 + 7x - 4$

Example 2: $12x^2 - 5x - 2$

Solution 2: $12x^2 - 5x - 2$ is first changed to $x^2 - 5x - 24$

$$(x - 8)(x + 3) \quad \text{Factor}$$

$$\left(x - \frac{8}{12}\right)\left(x + \frac{3}{12}\right) \quad \text{Divide each constant by 12, the original coefficient of } 12x^2$$

$$\left(x - \frac{2}{3}\right)\left(x + \frac{1}{4}\right) \quad \text{Simplify}$$

$$(3x - 2)(4x + 1) \quad \text{If there is fraction left after division, it becomes the coefficient of } x$$

Check using FOIL: $(3x - 2)(4x + 1) = 12x^2 - 5x - 24$

Factor by Grouping

- This method is similar to the AC method, but does not involve manipulating the initial equation
- We still need our “**multiply to this, add to this**” scenario, but with a minor tweak
- We are looking for **two numbers that add to the middle (b) term**, but that **multiply to the product of the a and c term**
- Then we **re-write the middle term** as the **sum of those two numbers**
- Now we **can group and factor**

Example 3: Factor $2x^2 + 7x - 4$ by grouping

Solution 3:

$$2x^2 + 7x - 4$$

$$2x^2 - 1x + 8x - 4$$

Identify two numbers that add to +7 and multiply to the product of the *a* and *c* term (2 and -4)
So add to +7 and multiply to -8

In this case it is: **-1 and 8**
Split the *b* term (+7x) into the (-1x + 8x)

$$x(2x - 1) + 4(2x - 1)$$

You need to be strategic to **factor out a common factor** so that you are **left with the same thing in the brackets**

$$(2x - 1)(x + 4)$$

Now each term had **(2x - 1) as a common factor** so factor it out.
You are left with $(2x - 1)(x + 4)$ to verify **imagine Waterbombing** the entire **(2x - 1) into the (x + 4)**, it will all become clearer.

Example 4: Factor $12x^2 - 5x - 2$

Solution 4:

$$12x^2 - 5x - 2$$

$$12x^2 - 8x + 3x - 2$$

Factor out a 4x

$$4x(3x - 2) + 1(3x - 2)$$

Factor out a +1

$$(3x - 2)(4x + 1)$$

Factor out the (3x - 2)

- Both Methods work great, it comes down to personal preference
- Enjoy!

Section 5.3 – Practice Problems**EMERGING LEVEL QUESTIONS**

Factor using Grouping or the AC Method

1. $2x^2 + 13x + 15$

2. $3x^2 + 8x + 4$

3. $10x^2 + 17x + 3$

4. $8y^2 - 18y + 9$

5. $21y^2 - 41y + 10$

6. $2y^2 - 7y + 5$

7. $20z^2 - 27z - 8$

8. $3z^2 - 20z - 63$

Factor Completely

9. $-3x^2 - x + 4$

10. $-2x^2 - 5xy - 2y^2$

11. $-6a^2 - 17ab + 3b^2$

12. $-4a^2b - 4ab^2 + 3b^3$

PROFICIENT LEVEL QUESTIONS

Factor Completely

13. $25x^2(a - 1)^3 - 5x(a - 1)^3 - 2(a - 1)^3$

14. $9 - 10x^2 + x^4$

15. $8x^4 + 19x^2 - 27$

16. $9x^4 - 145x^2 + 16$

Section 5.3 – Answer Key

1. $(x + 5)(2x + 3)$
2. $(x + 2)(3x + 2)$
3. $(2x + 3)(5x + 1)$
4. $(2y - 3)(4y - 3)$
5. $(3y - 5)(7y - 2)$
6. $(y - 1)(2y - 5)$
7. $(5z - 8)(4z + 1)$
8. $(z - 9)(3z + 7)$
9. $-(3x + 4)(x - 1)$
10. $-(x + 2y)(2x + y)$
11. $-(a + 3b)(6a - b)$
12. $-b(2a + 3b)(2a - b)$
13. $(a - 1)^3(5x - 2)(5x + 1)$
14. $(x + 3)(x - 3)(x + 1)(x - 1)$
15. $(8x^2 + 27)(x + 1)(x - 1)$
16. $(x + 4)(x - 4)(3x - 1)(3x + 1)$

Extra Work Space