Section 6.1b – Solving Right Angle Triangles

This booklet belongs to:______________________________Block: ______

- Solving triangles involves solving for **all three angles** and **all three sides**
- A quick reminder, **all three angles** in a triangle add up to $180°$
- At this level we only discuss **RIGHT ANGLE** triangles so we already know **one angle is** $90°$
- So, the **other two** must also **add to** $90°$

Solving Triangles

- Whenever we are solving triangle **we need at least 2 pieces** of information
- Either **2 sides** or **1 side and 1 angle**
- From there we can then solve for everything else

All of the rest of the information comes from **working with ratios**

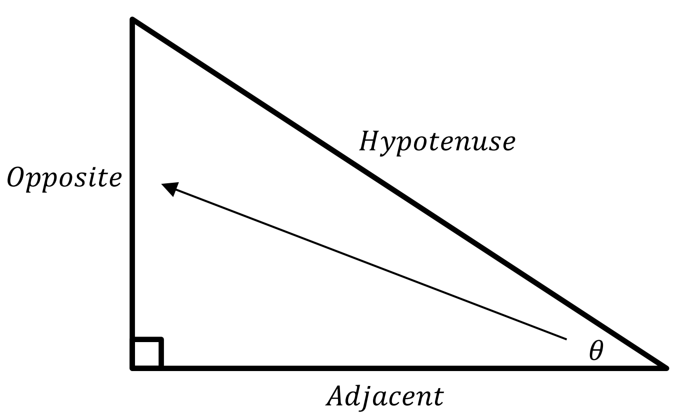
To help remember these ratios we think about these three words

SOH CAH TOA

They stand for:

$$\sin\theta = \frac{Opposite}{Hypotenuse} \qquad \cos\theta = \frac{Adjacent}{Hypotenuse} \qquad \tan\theta = \frac{Opposite}{Adjacent}$$

- With a right-angle triangle, **depending on what angle you want**, the sides get named differently



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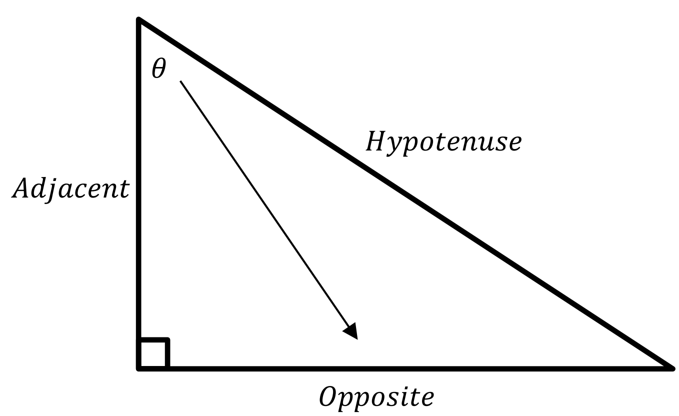
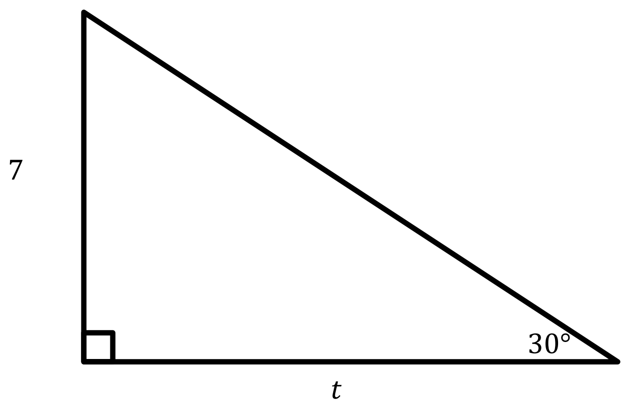


Different Solving Scenarios

An unknown Side – Using Tangent

- If you look at the triangle we have **an angle** and the **opposite side**
- We want the **adjacent side**
- So, we have **two letters of TOA**, so were using $\boldsymbol{TANGENT}$



- Using the **TOA Ratio** from before:

$$Tan(\theta) = \frac{Opposite}{Adjacent} \quad \rightarrow \quad Tan(30) = \frac{7}{t}$$

- Doing some Algebra:

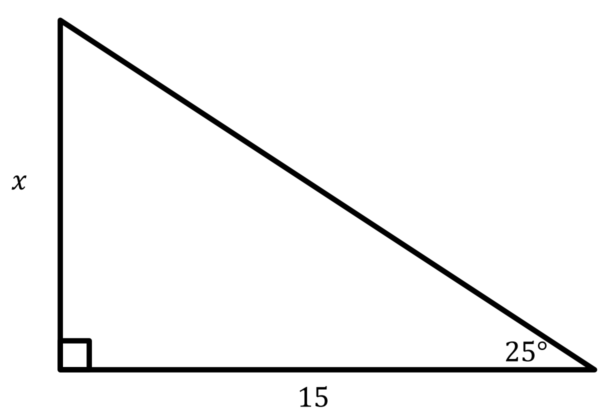
$$(t)Tan(30) = 7 \quad \rightarrow \quad t = \frac{7}{Tan(30)}$$

- Then we solve, you can simplify the $Tan(30)$ 1st

$$t = \frac{7}{0.5774} \quad \rightarrow \quad \boldsymbol{t = 12.1}$$

An unknown Side – Using Tangent

- If you look at the triangle we have **an angle** and the **adjacent side**
- We want the **opposite side**
- So we have **two letters of TOA**, so were using $\boldsymbol{TANGENT}$



- Using the **TOA Ratio** from before:

$$Tan(\theta) = \frac{Opposite}{Adjacent} \quad \rightarrow \quad Tan(25) = \frac{x}{15}$$

- Doing some Algebra:

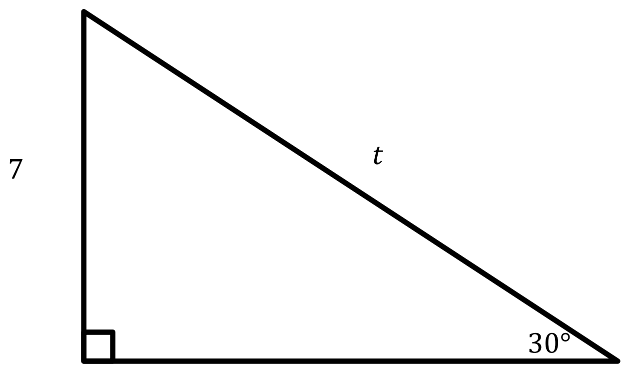
$$(15)Tan(25) = x$$

- Then we solve, you can simplify the $Tan(25)$ 1st

$$(15)(0.4663) = x \quad \rightarrow \quad \boldsymbol{x = 7.0}$$

An unknown Side – Using Sine

- If you look at the triangle we have **an angle** and the **opposite side**
- We want the **hypotenuse**
- So, we have **two letters of SOH**, so were using $\boldsymbol{SINE}$



- Using the **SOH Ratio** from before:

$$Sin(\theta) = \frac{Opposite}{Hypotenuse} \quad \rightarrow \quad Sin(30) = \frac{7}{t}$$

- Doing some Algebra:

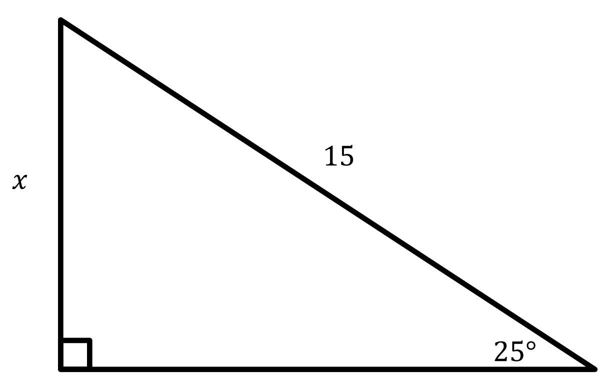
$$(t)Sin(30) = 7 \quad \rightarrow \quad t = \frac{7}{Sin(30)}$$

- Then we solve, you can simplify the $Sin(30)$ 1st

$$t = \frac{7}{0.5} \quad \rightarrow \quad \boldsymbol{t = 14}$$

An unknown Side – Using Sine

- If you look at the triangle we have **an angle** and the **hypotenuse**
- We want the **opposite side**
- So we have **two letters of SOH**, so were using $\boldsymbol{SINE}$



- Using the **SOH Ratio** from before:

$$Sin(\theta) = \frac{Opposite}{Hypotenuse} \quad \rightarrow \quad Sin(25) = \frac{x}{15}$$

- Doing some Algebra:

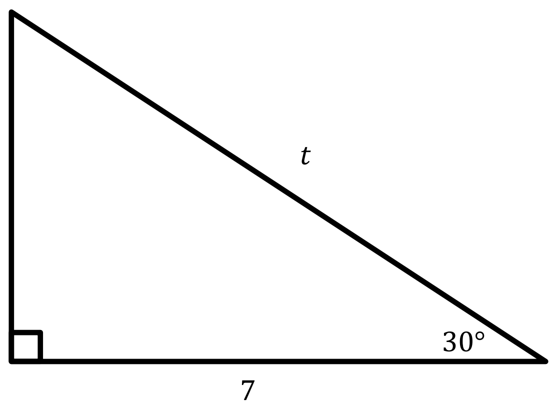
$$(15)Sin(25) = x$$

- Then we solve, you can simplify the $Sin(25)$ 1st

$$(15)(0.4226) = x \quad \rightarrow \quad \boldsymbol{x = 6.3}$$

An unknown Side – Using Cosine

- If you look at the triangle we have **an angle** and the **adjacent side**
- We want the **hypotenuse**
- So, we have **two letters of CAH**, so were using $\boldsymbol{COSINE}$



- Using the **CAH Ratio** from before:

$$Cos(\theta) = \frac{Adjacent}{Hypotenuse} \rightarrow Cos(30) = \frac{7}{t}$$

- Doing some Algebra:

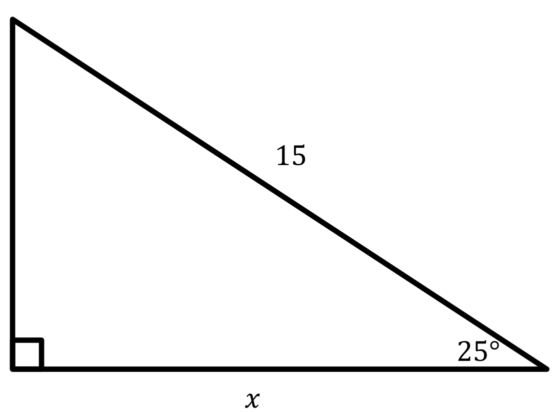
$$(t)Cos(30) = 7 \rightarrow t = \frac{7}{Cos(30)}$$

- Then we solve, you can simplify the $Cos(30)$ 1st

$$t = \frac{7}{0.8660} \rightarrow \boldsymbol{t = 8.1}$$

An unknown Side – Using Cosine

- If you look at the triangle we have **an angle** and the **hypotenuse**
- We want the **adjacent side**
- So, we have **two letters of CAH**, so were using $\boldsymbol{COSINE}$



- Using the **CAH Ratio** from before:

$$Cos(\theta) = \frac{Adjacent}{Hypotenuse} \rightarrow Cos(25) = \frac{x}{15}$$

- Doing some Algebra:

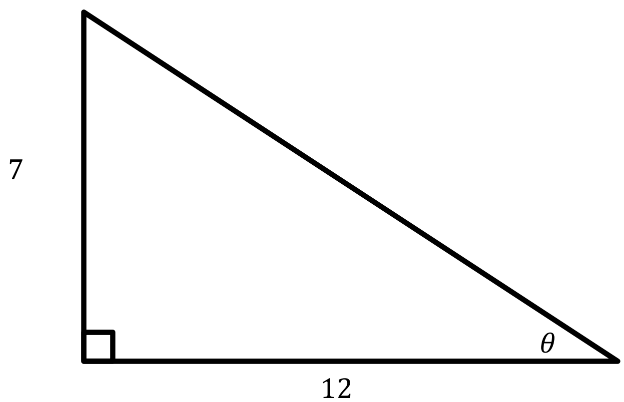
$$(15)Cos(25) = x$$

- Then we solve, you can simplify the $Cos(25)$ 1st

$$(15)(0.9063) = x \rightarrow \boldsymbol{x = 13.6}$$

An unknown Angle – Using Tangent

- If you look at the triangle we have **the opposite and adjacent sides**
- We want the **angle**
- So, we have **two letters of TOA**, so were using $\boldsymbol{TANGENT}$



- Using the **TOA Ratio** from before:

$Tan(\theta) = \frac{Opposite}{Adjacent} \rightarrow Tan(\theta) = \frac{7}{12}$

- Then we can simplify the $\frac{7}{12}$ 1st

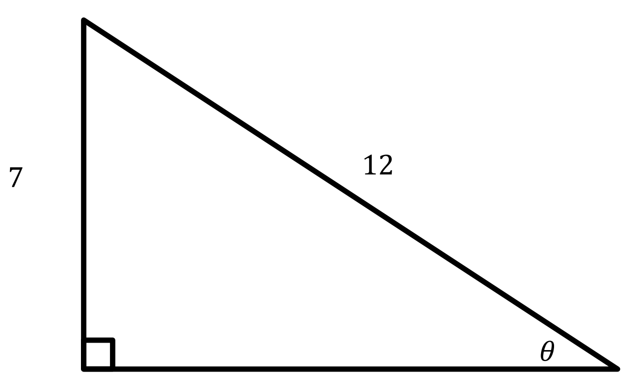
$Tan(\theta) = 0.5833$

- Then we use the inverse Tangent button

$\theta = Tan^{-1}(0.5833) \rightarrow \mathbf{30.3°}$

An unknown Angle – Using Sine

- If you look at the triangle we have **the opposite side and hypotenuse**
- We want the **angle**
- So, we have **two letters of SOH**, so were using $\boldsymbol{SINE}$



- Using the **SOH Ratio** from before:

$Sin(\theta) = \frac{Opposite}{Hypotenuse} \rightarrow Sin(\theta) = \frac{7}{12}$

- Then we can simplify the $\frac{7}{12}$ 1st

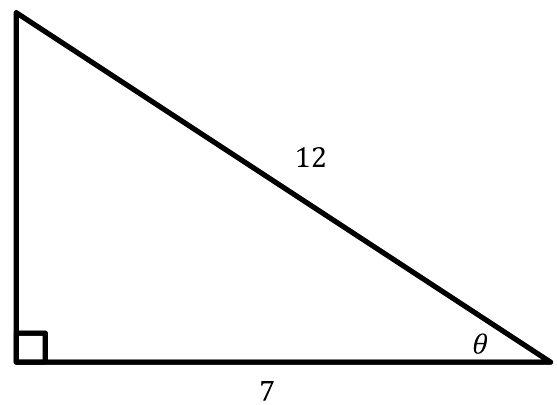
$Sin(\theta) = 0.5833$

- Then we use the inverse Sine button

$\theta = Sin^{-1}(0.5833) \rightarrow \mathbf{35.7°}$

An unknown Angle – Using Cosine

- If you look at the triangle we have **the adjacent side and hypotenuse**
- We want the **angle**
- So, we have **two letters of CAH**, so were using $\boldsymbol{COSINE}$



- Using the **CAH Ratio** from before:

$$Cos(\theta) = \frac{Adjacent}{Hypotenuse} \quad \rightarrow \quad Cos(\theta) = \frac{7}{12}$$

- Then we can simplify the $\frac{7}{12}$ 1st

$Cos(\theta) = 0.5833$

- Then we use the inverse Sine button

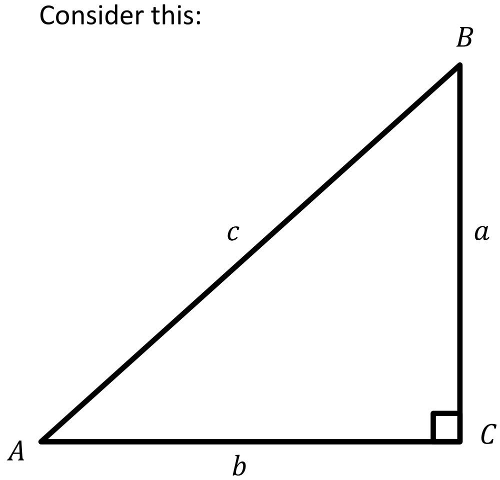
$$\theta = Cos^{-1}(0.5833) \quad \rightarrow \quad \mathbf{54.3°}$$

Relationship between Sine, Cosine, and Tangent

- Now that we have seen the ratio and angle relationship between Sine, Cosine, and Tangent, let's look at how they relate to one another.

Sine and Cosine

Consider this:



- **All angles** in a **triangle add up to** $\mathbf{180°}$
- Since $\angle \boldsymbol{C} = \mathbf{90°}$, it means that $\angle \boldsymbol{A} + \angle \boldsymbol{B} = \mathbf{90°}$

This gives us our first look at a trigonometric identity.

If: $\angle \boldsymbol{A} + \angle \boldsymbol{B} = \mathbf{90°}$

Then: $\angle \boldsymbol{A} = \mathbf{90°} - \angle \boldsymbol{B}$ and $\angle \boldsymbol{B} = \angle \boldsymbol{A} - \mathbf{90°}$

From our SOH CAH TOA ratios we look at, and considering the triangle on the left:

$$\mathbf{sin}\, \boldsymbol{A} = \left(\frac{\boldsymbol{a}}{\boldsymbol{c}}\right) = \mathbf{cos}\, \boldsymbol{B}$$

$$\mathbf{cos}\, \boldsymbol{A} = \left(\frac{\boldsymbol{b}}{\boldsymbol{c}}\right) = \mathbf{sin}\, \boldsymbol{B}$$

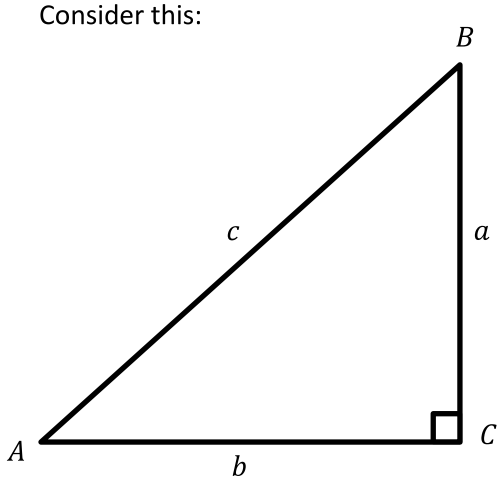
So, $\sin\theta = \cos(90° - \theta)$ *and* $\cos\theta = \sin(90° - \theta)$

Example: $\sin 30° = \cos 60°$ $\cos 30° = \sin 60°$

$\sin 55° = \cos 35°$ $\cos 55° = \sin 35°$

Tangent

Consider this:



From our SOH CAH TOA ratios we look at, and considering the triangle on the left:

$$\sin A = \left(\frac{a}{c}\right) \quad \cos A = \left(\frac{b}{c}\right) \quad \tan A = \left(\frac{a}{b}\right)$$

So, we get a relationship that can be represented as follows:

$$\frac{\sin A}{\cos A} = \frac{\left(\frac{a}{c}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{c}\right) \cdot \left(\frac{c}{b}\right) = \frac{a}{b} = \tan A$$

So, we get another trigonometric identity:

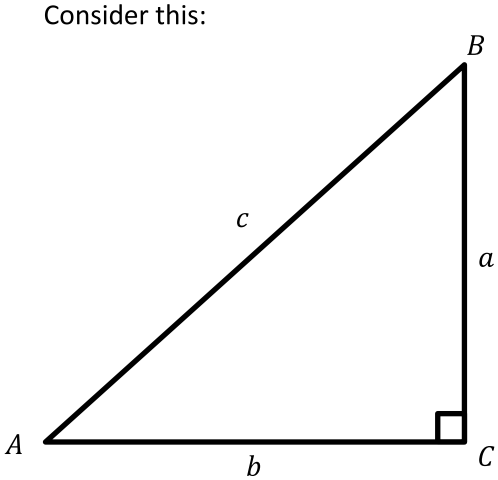
$$\frac{\sin A}{\cos A} = \tan A$$

Example: $\frac{\sin 30°}{\cos 30°} = \tan 30°$

Sine² and Cosine²

- We will look at two more identities that will come in handy in the years ahead.
- It involves our trigonometric ratios and we tie that to the Pythagorean Theorem

Consider this:



From Pythagoras we get:

$$a^2 + b^2 = c^2$$

And comparing that to our SOH CAH TOA ratios, we have:

$$\sin A = \frac{a}{c} \quad and \quad \cos A = \frac{b}{c}$$

So, doing a bit of algebra:

$$c \sin A = a \quad and \quad c \cos A = b$$

Square both sides:

$$c^2 \sin^2 A = a^2 \quad and \quad c^2 \cos^2 A = b^2$$

From there: (→ $c^2 \sin^2 A = a^2$ and $c^2 \cos^2 A = b^2$)
We go:

If:

Substitution for a^2 and b^2

Divide both sides by c^2

$$a^2 + b^2 = c^2 \quad \rightarrow \quad c^2 \sin^2 A + c^2 \cos^2 A = c^2 \quad \rightarrow \quad \mathbf{\sin^2 A + \cos^2 A = 1}$$

Tangent^2

- The last identity we will see comes directly from the one above

If: $\sin^2 A + \cos^2 A = 1$ Note: $\sin^2 A = (\sin A)^2$

Then: If we divide all terms by $\cos^2 A$ we get:

$$\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A}$$

This simplifies to:

$$\mathbf{\tan^2 A + 1 = \frac{1}{\cos^2 A}}$$

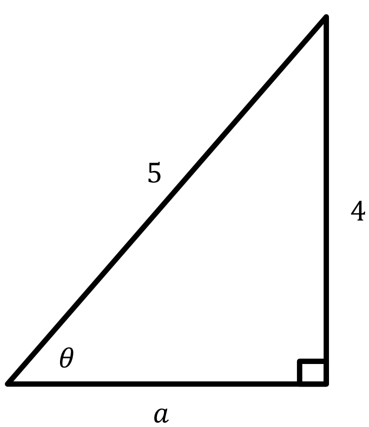
So, for this grade, we have **5 Trigonometric Identities**

1. $\sin\theta = \cos(90° - \theta)$	2. $\cos\theta = \sin(90° - \theta)$	3. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
4. $\sin^2\theta + \cos^2\theta = 1$	5. $\tan^2\theta + 1 = \frac{1}{\cos^2 A}$	

With this information we can solve for the missing information in a right-angle triangle in a number of ways

<u>Example 1:</u> Given $\sin\theta = \frac{4}{5}$ where θ is an acute angle. Find $\cos\theta$ and $\tan\theta$

<u>Solution 1:</u> Draw the triangle first.



In order to solve:

$$\cos\theta = \frac{a}{5} \quad and \quad \tan\theta = \frac{4}{a}$$

We need to know a.

Pythagorean Theorem is our best bet.

$$a^2 + b^2 = c^2 \quad \rightarrow \quad a^2 + 4^2 = 5^2$$

$$a^2 + 16 = 25 \quad \rightarrow \quad a^2 = 25 - 16$$

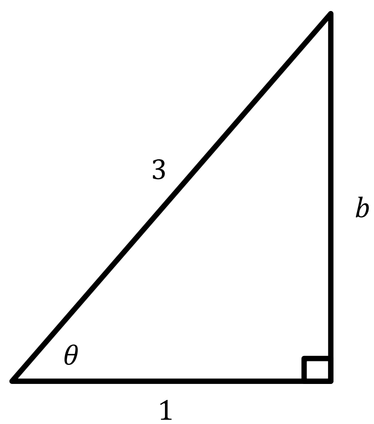
$$a^2 = 9 \quad \rightarrow \quad a = 3$$

Therefore,

$$\boldsymbol{\cos\theta = \frac{3}{5} \quad and \quad \tan\theta = \frac{4}{3}}$$

Example 2: Given $\cos\theta = \frac{1}{3}$ find both $\sin\theta$ and $\tan\theta$

Solution 2: Draw the triangle first.



In order to solve:

$$\sin\theta = \frac{b}{3} \quad and \quad \tan\theta = \frac{b}{1}$$

We need to know b.

Pythagorean Theorem is our best bet.

$$a^2 + b^2 = c^2 \quad \rightarrow \quad 1^2 + b^2 = 3^2$$

$$1 + b^2 = 9 \quad \rightarrow \quad b^2 = 9 - 1$$

$$b^2 = 8 \quad \rightarrow \quad b = \sqrt{8}$$

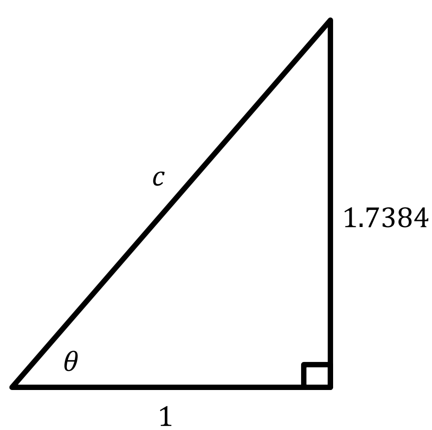
Therefore,

$$\mathbf{\sin\theta = \frac{\sqrt{8}}{3} \quad and \quad \tan\theta = \frac{\sqrt{8}}{1} = \sqrt{8}}$$

- It works the same when given numerical values. Just consider the given ratio and use 1 as a denominator.

Example 3: Given $\tan\theta = 1.7384$ find both $\sin\theta$ and $\cos\theta$

Solution 3: Draw the triangle first.



In order to solve:

$$\sin\theta = \frac{1.7384}{c} \quad and \quad \cos\theta = \frac{1}{c}$$

We need to know c.

Pythagorean Theorem is our best bet.

$$a^2 + b^2 = c^2 \quad \rightarrow \quad 1.7384^2 + 1^2 = c^2$$

$$3.0220 + 1 = c^2 \quad \rightarrow \quad c^2 = 4.0220$$

$$c = \sqrt{4.0220} = 2.006 = 2$$

Therefore,

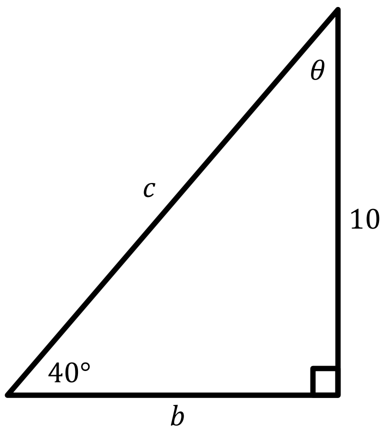
$$\mathbf{\sin\theta = \frac{1.7384}{2} = 0.8668 \quad and \quad \cos\theta = \frac{1}{2}}$$

Section 6.1b – Practice Problems

EMERGING LEVEL QUESTIONS

Solve for all the missing information. Round to the nearest tenth if necessary. (Drawings are not to Scale)

1.

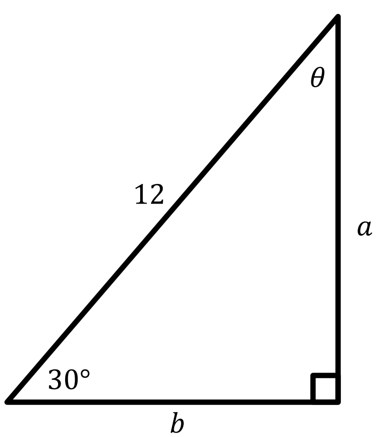


$b =$

$c =$

$\theta =$

2.

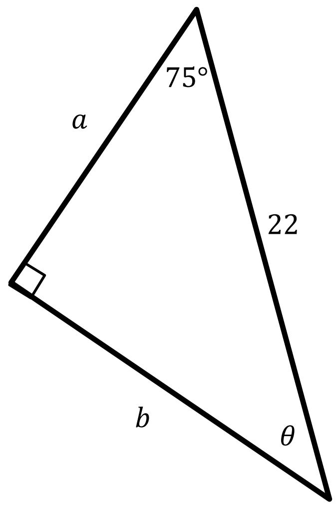


$b =$

$a =$

$\theta =$

3.

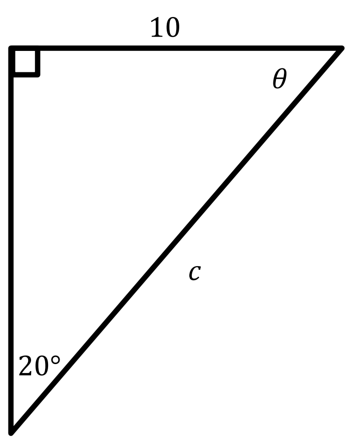


$b =$

$a =$

$\theta =$

4.

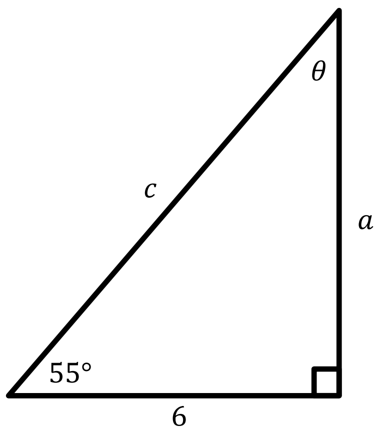


$b =$

$c =$

$\theta =$

5.

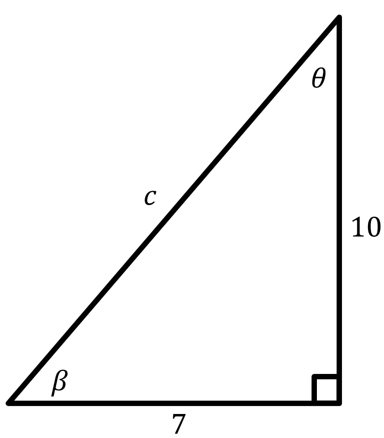


$a =$

$c =$

$\theta =$

6.



$\beta =$

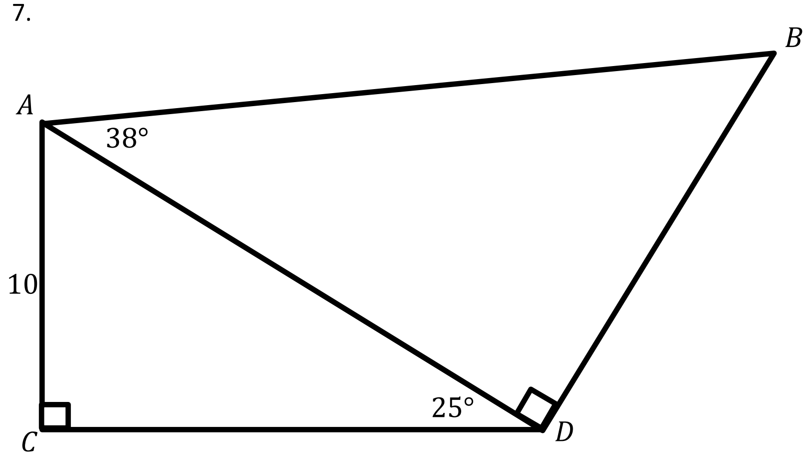
$c =$

$\theta =$

PROFICIENT LEVEL QUESTIONS

Find the length of side AB

7.



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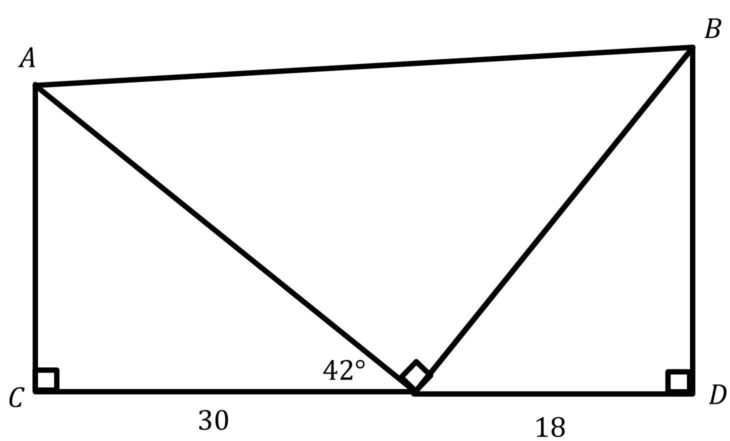
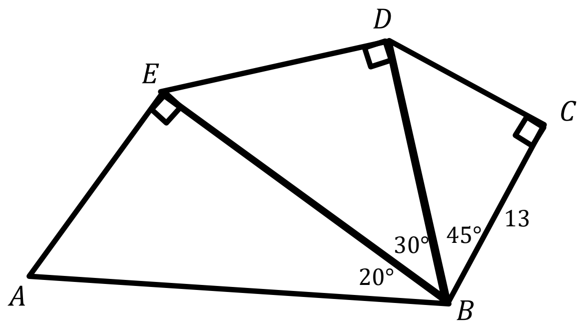
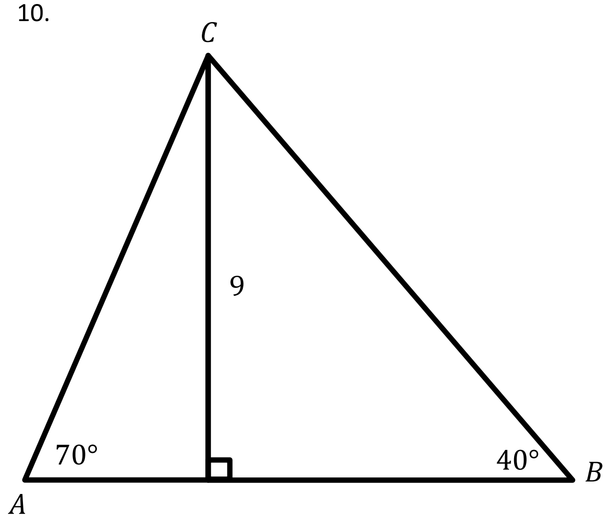


9.



10.



Find the exact value of the remaining trigonometric functions of the acute angle θ.

11. $\sin\theta = \frac{6}{11}$

12. $\tan\theta = \frac{4}{3}$

13. $\cos\theta = \frac{15}{17}$

14. $\sin\theta = \frac{\sqrt{3}}{2}$

15. $\cos\theta = \frac{1}{3}$

16. $\tan\theta = \frac{6}{7}$

17. $\sin\theta = \frac{3}{7}$

18. $\cos\theta = \frac{\sqrt{5}}{3}$

19. $\sin\theta = 0.4896$

20. $\cos\theta = 0.7942$

EXTENDING LEVEL QUESTIONS

21. If a triangle has a value of $\sin 30°$, what is the cosine value in the same triangle?

22. Solve

$$\sin^2 45° + \cos^2 45° = ?$$

23. Write $\tan 65°$ in terms of sine and cosine.

24. Write $\sin^2\theta + \cos^2\theta = 1$ in terms of tangent and cosine only.

Answer Key – Section 6.1b

1. $b = 11.9, \quad c = 15.6, \quad \theta = 50°$
2. $b = \sqrt{108}, \quad a = 6, \quad \theta = 60°$
3. $b = 21.3, \quad a = 5.7, \quad \theta = 15°$
4. $b = 27.5, \quad c = 29.2, \quad \theta = 70°$
5. $a = 8.6, \quad c = 10.5, \quad \theta = 35°$
6. $\beta = 55°, \quad c = \sqrt{149}, \theta = 35°$
7. $AB = 30$
8. $AB = 48.5$
9. $AB = 22.6$
10. $AB = 14$
11. $\tan\theta = \frac{6}{\sqrt{85}} \quad \cos\theta = \frac{\sqrt{85}}{11}$
12. $\sin\theta = \frac{4}{5} \quad \cos\theta = \frac{3}{5}$
13. $\sin\theta = \frac{8}{17} \quad \tan\theta = \frac{8}{15}$
14. $\cos\theta = \frac{1}{2} \quad \tan\theta = \sqrt{3}$
15. $\sin\theta = \frac{\sqrt{8}}{3} \quad \tan\theta = \sqrt{8}$
16. $\sin\theta = \frac{6}{\sqrt{85}} \quad \cos\theta = \frac{7}{\sqrt{85}}$
17. $\cos\theta = \frac{\sqrt{40}}{7} \quad \tan\theta = \frac{3}{\sqrt{40}}$
18. $\sin\theta = \frac{2}{3} \quad \tan\theta = \frac{2}{\sqrt{5}}$
19. $\cos\theta = 0.8719, \quad \tan\theta = 0.5615$
20. $\sin\theta = 0.6077, \quad \tan\theta = 0.7651$
21. $60°$
22. 1
23. $\frac{\sin 65°}{\cos 65°}$
24. $\tan^2\theta + 1 = \frac{1}{\cos^2\theta}$

Extra Work Space