

Section 6.1c – Special Angles

This booklet belongs to: _____ **Block:** _____

- Two triangles in trigonometry are especially significant, we can calculate them exactly
- They are the $45^\circ - 45^\circ - 90^\circ$ triangle and the $30^\circ - 60^\circ - 90^\circ$ triangle.

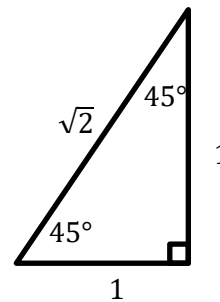
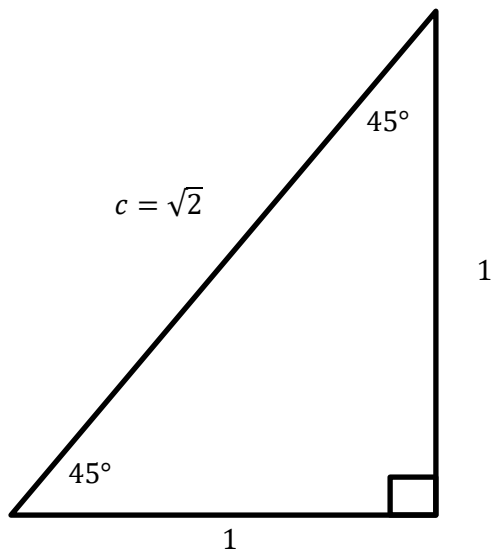
The $45^\circ - 45^\circ - 90^\circ$ Triangle

- Since the triangle has two equal angles, it is an isosceles
- Since trigonometric function are based on ratios we can use any numbers, use 1 for simplicity
- By Pythagoras' Theorem:

$$c^2 = a^2 + b^2 = 1^2 + 1^2 \quad \rightarrow \quad c = \sqrt{2}$$

Therefore,

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \quad \cos 45^\circ = \frac{1}{\sqrt{2}} \quad \tan 45^\circ = \frac{1}{1} = 1$$



The 30° – 60° – 90° Triangle

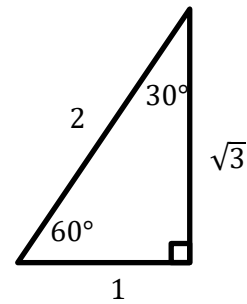
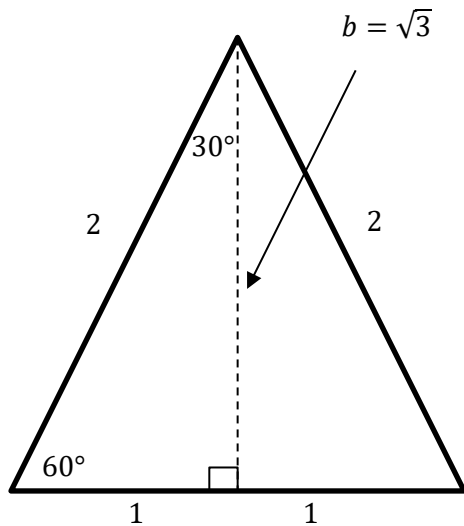
- Consider an equilateral triangle with all sides equal to 2
- Draw an altitude from a base to split the opposite 60 in half
- By Pythagoras' Theorem:

$$c^2 - a^2 = b^2 \quad \rightarrow \quad 2^2 - 1^2 = b^2 \quad \rightarrow \quad b = \sqrt{3}$$

Therefore,

$$\sin 30^\circ = \frac{1}{2} \quad \cos 30^\circ = \frac{\sqrt{3}}{2} \quad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cos 60^\circ = \frac{1}{2} \quad \tan 60^\circ = \sqrt{3}$$



Summary of Special Angles

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Section 6.1c – Practice ProblemsFind the exact value of each expression if $\theta = 30^\circ$

1. $\sin \theta$

2. $\cos \theta$

3. $\tan \theta$

4. $\sin 2\theta$

5. $\cos 2\theta$

6. $\tan 2\theta$

Find the exact value of each expression if $\theta = 45^\circ$

7. $\sin \theta$

8. $\cos \theta$

9. $\tan \theta$

10. $2\sin \theta$

11. $2\cos \theta$

12. $2\tan \theta$

Find the exact value of each expression if $\theta = 60^\circ$

13. $\sin \theta$

14. $\cos \theta$

15. $\tan \theta$

16. $\sin \frac{\theta}{2}$

17. $\cos \frac{\theta}{2}$

18. $\tan \frac{\theta}{2}$

Answer Key – Section 6.1c

1. $\frac{1}{2}$
2. $\frac{\sqrt{3}}{2}$
3. $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
4. $\frac{\sqrt{3}}{2}$
5. $\frac{1}{2}$
6. $\sqrt{3}$
7. $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$
8. $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$
9. 1
10. $\sqrt{2}$
11. $\sqrt{2}$
12. 2
13. $\frac{\sqrt{3}}{2}$
14. $\frac{1}{2}$
15. $\sqrt{3}$
16. $\frac{1}{2}$
17. $\frac{\sqrt{3}}{2}$
18. $\frac{1}{\sqrt{3}}$

Extra Work Space